A1G1) **1. PRÜFUNG** 12. DEZ. 2024.

(1) $z^2+1-i=0 \ \vee \ |z-2|>2 \hookrightarrow |z-2|=2$ Kreis, MP$(2;0)$, $r=2$ (2)

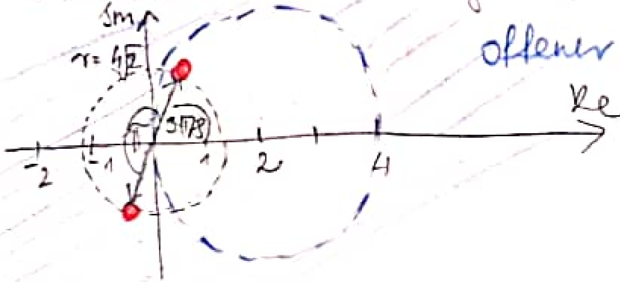
$z^2=-1+i=\sqrt{2}\cdot\left(\cos\frac{3\pi}{4}+i\cdot\sin\frac{3\pi}{4}\right)$ (1)

$z=\sqrt[4]{2}\cdot\left(\cos\frac{\frac{3\pi}{4}+k2\pi}{2}+i\cdot\sin\frac{\frac{3\pi}{4}+k2\pi}{2}\right)$ $k=0,1$ (1)

$z_1=\sqrt[4]{2}\cdot\left(\cos\frac{3\pi}{8}+i\cdot\sin\frac{3\pi}{8}\right)$

$z_2=\sqrt[4]{2}\cdot\left(\cos\frac{11\pi}{8}+i\cdot\sin\frac{11\pi}{8}\right)$ } zwei Punkte (2)

Lös.: e Union v Punktmengen:



(2)

(2) $f(x)=x\cdot\operatorname{arctg}(x)$

$D_f=\mathbb{R} \Rightarrow$ keine USS-en $\Rightarrow$ keine senkrechte Asymptoten (1)

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to\infty} f(x) = +\infty \notin \mathbb{R} \Rightarrow$ keine waagerechte Asymptoten (1)

f gerade; $\text{"}\infty\cdot\frac{\pi}{2}\text{"}$

Schräge (schiefe) Asymptoten?

$\lim\limits_{x\to-\infty}\frac{f(x)}{x} = \lim\limits_{-\infty}\operatorname{arctg} x = \frac{-\pi}{2} = m_1$ (1)

$\lim\limits_{x\to+\infty}\frac{f(x)}{x} = \lim\limits_{+\infty}\operatorname{arctg}(x) = \frac{+\pi}{2} = m_2$ (1)

$b_1=\lim\limits_{x\to-\infty} x\cdot\operatorname{arctg}x+\frac{\pi}{2}x = \lim\limits_{-\infty} x\cdot\left(\operatorname{arctg}x+\frac{\pi}{2}\right) =$ $\text{"}-\infty\cdot 0\text{"}$

$=\lim\limits_{-\infty}\frac{\operatorname{arctg}x+\frac{\pi}{2}}{\frac{1}{x}} = -1$ $\text{"}\frac{0}{0}\text{"}$ L'Hospital $\boxed{?\ \lim\limits_{-\infty}\frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}}=-1}$ (1)

$b_2=\lim\limits_{x\to\infty} x\left(\operatorname{arctg}x-\frac{\pi}{2}\right) = \lim\limits_{\infty}\frac{\operatorname{arctg}x-\frac{\pi}{2}}{\frac{1}{x}} = -1$ $\text{"}\infty\cdot 0\text{"}$ $\text{"}\frac{0}{0}\text{"}$ (1)

$\boxed{?\ \lim\limits_{\infty}\frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}}=-1}$ L'Hospital-Satz

Zwei schiefe Asymptoten:

in $-\infty$: $y=-\frac{\pi}{2}\cdot x-1$ (1)

in $+\infty$: $y=\frac{\pi}{2}\cdot x-1$ (1)

(3) $y\operatorname{tg}(xy)-\frac{x+y}{x^2}+\ln\frac{y}{\pi x}+\ln 4=-1$ $P_0\left(1;\frac{\pi}{4}\right)$ einsetzen

$\cancel{\frac{\pi}{4}\cdot 1}-1-\cancel{\frac{\pi}{4}}+\cancel{\ln\frac{1}{4}}+\cancel{\ln 4}=-1$ (2)

$-1\equiv-1$ ✓ $P_0\in$ Kurve

$y\cdot\operatorname{tg}(xy)-\frac{1}{x}-\frac{y}{x^2}+\ln y-\ln x-\ln\pi+\ln 4=-1$

Ableitung

$y'\cdot\operatorname{tg}(xy)+y\cdot\frac{y+x\cdot y'}{\cos^2(xy)}+\frac{1}{x^2}-\frac{y'x^2-2yx}{x^4}+\frac{y'}{y}-\frac{1}{x}=0$ (3)

$y'\cdot\left(\operatorname{tg}(xy)+\frac{xy}{\cos^2(xy)}-\frac{1}{x^2}+\frac{1}{y}\right)=\frac{1}{x}-\frac{2y}{x^3}-\frac{1}{x^2}-\frac{y^2}{\cos^2(xy)}$

in P_0: $y_0'\cdot\left(\cancel{1}+\frac{\pi/4}{1/2}-\cancel{1}+\frac{4}{\pi}\right)=\cancel{1}-\frac{\pi}{2}-\cancel{1}-\frac{\pi^2/16}{1/2}$

$\Rightarrow y_0'=\frac{-\frac{\pi}{2}-\frac{\pi^2}{8}}{\frac{\pi}{2}+\frac{4}{\pi}}=\frac{-4\pi-\pi^2}{4\pi+\frac{32}{\pi}} \Rightarrow y=\frac{-4\pi-\pi^2}{4\pi+\frac{32}{\pi}}\cdot(x-1)+\frac{\pi}{4}$ (3)

4 a) $\int \underset{u}{x} \cdot \underset{v'}{\sin x}\, dx = -x\cdot\cos x + \int \cos x\, dx = -x\cdot\cos x + \sin x + C$

$u' = 1,\ v = -\cos x$ (2) $C \in \mathbb{R}$

b) $\int \frac{x^3+5x+10}{x^3+2x-12}\,dx = \int \frac{x^3+2x-12+3x+22}{x^3+2x-12}\,dx =$

$= \int 1 + \frac{3x+22}{x^3+2x-12}\,dx$ (1) $= \int 1 + \frac{2}{x-2} - \frac{2x+5}{x^2+2x+6}\,dx =$ (1)

$= \int 1 + \frac{2}{x-2} - \frac{2x+2}{x^2+2x+6} - \frac{3}{\left(\frac{x+1}{\sqrt5}\right)^2+1}\cdot\frac{1}{5}\,dx =$ (1)

$= \boxed{x + 2\cdot\ln|x-2| - \ln|x^2+2x+6| - \frac{3}{5}\cdot\frac{\operatorname{arctg}\frac{x+1}{\sqrt5}}{\frac{1}{\sqrt5}} + C \quad C\in\mathbb{R}}$ (2)

$x^3+0x^2+2x-12 : (x-2) = \underbrace{x^2+2x+6}_{(x+1)^2+5}$ (1)

x^3-2x^2

$2x^2+2x$

$2x^2-4x$

$6x-12$

$6x-12$

0

$\frac{3x+22}{x^3+2x-12} = \frac{a}{x-2} + \frac{bx+c}{x^2+2x+6} = \frac{(a+b)x^2+(2a-2b+c)x+6a-2c}{x^3+2x-12}$

$\begin{cases} a+b=0 \\ 2a-2b+c=3 \\ 6a-2c=22 \end{cases}$ $\boxed{b=-a}$

$4a+c=3$

$3a-c=11$

$7a = 14$

$a = 2$

$b = -2$

$c = 3-4\cdot2 = -5$ (2)

5 $f(x) = 1+4\cdot x^{\frac32}$ $x\in\left[0; \frac29\right]$

$f'(x) = 6\cdot\sqrt{x}$ $(f'(x))^2 = 36x$ (1)

$B = \int_0^{2/9} \sqrt{1+36\cdot x}\,dx$ (1) $= \left[\frac{(1+36x)^{\frac32}}{\frac32\cdot36}\right]_0^{2/9} =$ (1)

$= \frac{1}{3\cdot18}\cdot\left(9^{\frac32} - 1^{3/2}\right) = \frac{1}{3\cdot18}\cdot(\overset{26}{27-1}) = \frac{13}{27}$ (1)

$V = \pi\cdot\int_0^{2/9}\left(1+4\cdot x^{\frac32}\right)^2 dx$ (1) $= \pi\cdot\int_0^{2/9} 1+8\cdot x^{\frac32}+16\cdot x^3\,dx =$ (1)

$= \pi\cdot\left[x + 8\cdot\frac25\cdot x^{\frac52} + 4\cdot x^4\right]_0^{\frac29} =$ (1)

$= \pi\cdot\left(\frac29 + \frac{16}{5}\cdot\left(\frac{\sqrt2}{3}\right)^5 + \frac{64}{9^4}\right) = \ldots$ (1)