A1G1 | 2. PRÜFUNG | 2024.

(1) $2x^4+8x^3+16x^2+22x+12 = 2(x^4+4x^3+8x^2+11x+6)$

$\pm1, \pm2, \pm3, \pm6$ (1p)

$x_1=-1$ Wurzel? (1p)

$x_2=-2$ " } auch $\Rightarrow (x+1)(x+2) = x^2+3x+2$ ausheben

$$
\begin{array}{l}
x^4+4x^3+8x^2+11x+6 : (x^2+3x+2) = \underbrace{x^2+x+3} \quad (3p) \\
\underline{x^4+3x^3+2x^2} \\
\quad x^3+6x^2+11x \\
\quad \underline{x^3+3x^2+2x} \\
\qquad 3x^2+9x+6 \\
\qquad \underline{3x^2+9x+6} \\
\qquad\qquad 0
\end{array}
$$

$x_{3,4} = \frac{-1\pm\sqrt{-11}}{2} = \frac{-1}{2}\pm\frac{\sqrt{11}}{2}\cdot i$ (1p)

über $\mathbb{R}$: $2(x+1)(x+2)(x^2+x+3)$

$\mathbb{C}$: $2(x+1)(x+2)(x+\frac{1}{2}+\frac{\sqrt{11}}{2}i)(x+\frac{1}{2}-\frac{\sqrt{11}}{2}i)$ } (2p)

(2) $a_n = \frac{3-n}{2n+1}$ $\quad \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{\frac{3}{n}-1}{2+\frac{1}{n}} = \frac{0-1}{2+0} = \frac{-1}{2}$ (2p)

a_n konvergent $\Rightarrow$ auch beschränkt

Monotonie:

$a_{n+1} = \frac{2-n}{2n+3} \overset{?}{\leq} \frac{3-n}{2n+1} = a_n$

$\Leftrightarrow (2-n)(2n+1) \overset{?}{\leq} (3-n)(2n+3)$

$\Leftrightarrow -2n^2+3n+2 \overset{?}{\leq} -2n^2+3n+9$ (2p)

$\Leftrightarrow 0 \overset{?}{\leq} 7 \quad \forall n\in\mathbb{N} \quad 0<7 \Leftrightarrow a_{n+1}<a_n$

a_n echt monoton fallend (1p)

$-\frac{1}{2} \quad \ldots \quad a_4 \quad a_3 \quad a_2 \quad a_1 = \frac{2}{3}$

$\nexists$ Minimum, $\inf a_n = \frac{-1}{2}$] (1p)

$\forall n\in\mathbb{N} \quad -\frac{1}{2} < a_n \leq \frac{2}{3}$ (1p) $\quad \max a_n = \sup a_n = a_1 = \frac{2}{3}$ (1p)

Beschränktheit

(3) $f(x) = \ln(x^2-5)$ $\quad$ 1) $D_f = \mathbb{R}\setminus[-\sqrt5;\sqrt5]$

2) Nullstelle: $x^2=6 \Leftrightarrow x_1=-\sqrt6,\ x_2=\sqrt6$ (1p)

3) f ist gerade $(x^2=(-x)^2 \ \forall x\in D_f)$

nicht periodisch

4) $\lim\limits_{x\to-\sqrt5^-} f(x) = \lim\limits_{x\to\sqrt5^+} \ln(\underbrace{x^2-5}_{\to 0+}) = -\infty$ $\quad$ $x=-\sqrt5$, $x=\sqrt5$ } senkrechte Asympt.

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to\infty} \ln(\underbrace{x^2-5}_{\to\infty}) = \infty$ keine waagr. Asymptoten

Schräge As.? $\lim\limits_{\pm\infty} \frac{\ln(x^2-5)}{x}$

$\overset{?}{=} \lim\limits_{\pm\infty} \frac{\frac{2x}{x^2-5}}{1} = 0 \Rightarrow$ keine schräge As. (2p)

5) $f'(x) = \frac{2x}{x^2-5}$

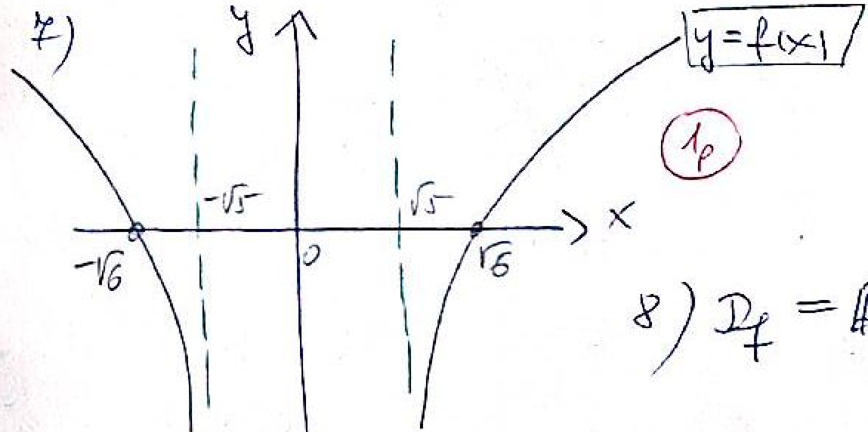
	$x<-\sqrt5$	$[-\sqrt5;\sqrt5]$	$x>+\sqrt5$
f'	$-$		$+$
f	↗		↘

keine Extrema (2p)

6) $f''(x) = \frac{2(x^2-5)-(2x)^2}{(x^2-5)^2} = \frac{-2x^2-10}{(x^2-5)^2} < 0$ auf D_f

$\Rightarrow f$ überall auf D_f konkav (1p)

7)



(1p)

8) $D_f = \mathbb{R}$ (1p)

④

a) $\int\sqrt{1+4x^2}\,dx = \int\frac{1}{2}ch^2(t)\,dt = \frac{1}{4}\cdot\int ch(2t)+1\,dt$ ⊛

$\left(\begin{array}{l} 2x = sh\,t \quad t\in\mathbb{R} \\ x = \frac{sh\,t}{2} \Rightarrow dx = \frac{ch\,t}{2}\cdot dt \end{array}\right)$ 1p

⊛ $= \frac{1}{4}\cdot\left(\frac{sh\,2t}{2} + t\right) + c =$

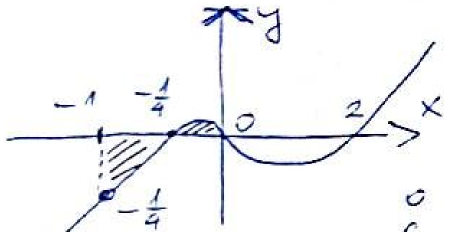
$= \frac{1}{4}\cdot\left(\frac{sh(2\cdot arsh\,2x)}{2} + arsh\,2x\right) + c$ 1p $\quad c\in\mathbb{R}$

1p

b) $y = 8x^3 - 14x^2 - 4x = 2x(4x^2 - 7x - 2)$

Nullstellen $\begin{cases} x_1 = 0 & x_{2,3} = \frac{7\pm\sqrt{49+32}}{8} \\ x_2 = -\frac{1}{4} & x_3 = 2 \end{cases}$ (9) 1p

$x\in[-1;0] \Rightarrow$ Flächeninhalt...



1p + 1p

$F = \left|\int_{-1}^{-\frac{1}{4}} f(x)\,dx\right| + \int_{-\frac{1}{4}}^{0} f(x)\,dx =$ 1p

$= \left|\left[2x^4 - \frac{14}{3}x^3 - 2x^2\right]_{-1}^{-\frac{1}{4}}\right| + \left[\dots\right]_{-\frac{1}{4}}^{0} =$ 1p

$= \left|\frac{2}{4^4} + \frac{14}{3\cdot 4^3} - \frac{1}{8} - \left(\cancel{2} + \frac{14}{3} - \cancel{2}\right)\right| + \left(0 - \left(\frac{2}{4^4} + \frac{14}{3\cdot 4^3} - \frac{1}{8}\right)\right) =$ 1p

$= \left|\frac{1/2}{4^3} + \frac{14/3}{4^3} - \frac{8}{4^3} - \frac{14}{3}\right| + \left(\frac{-1/2}{4^3} - \frac{14/3}{4^3} + \frac{8}{4^3}\right)$

⑤ $A\begin{bmatrix}1\\1\\1\end{bmatrix}$ $B\begin{bmatrix}-1\\0\\1\end{bmatrix}$ $C\begin{bmatrix}2\\2\\0\end{bmatrix}$ $D\begin{bmatrix}-1\\2\\2\end{bmatrix}$

a) Gl. v. Ebene: $P_0 = A$, $\underline{n} = \vec{AB}\times\vec{AC}$

$\underline{n} = \begin{bmatrix}-2\\-1\\0\end{bmatrix}\times\begin{bmatrix}1\\1\\-1\end{bmatrix} = \begin{vmatrix} i & j & k \\ -2 & -1 & 0 \\ 1 & 1 & -1 \end{vmatrix} = \begin{bmatrix}+(1-0)\\-(2-0)\\+(-2+1)\end{bmatrix} = \begin{bmatrix}1\\-2\\-1\end{bmatrix}$ 1

$\left\langle\begin{bmatrix}x-1\\y-1\\z-1\end{bmatrix};\begin{bmatrix}1\\-2\\-1\end{bmatrix}\right\rangle = x - 2x - z\,\cancel{-1} + 2\,\cancel{+1} = 0$ 1

2

$d(D;\text{Ebene}) = \left|\frac{-1-2\cdot 2-\cancel{2}+\cancel{2}}{\sqrt{1+4+1}}\right| = \frac{5}{\sqrt{6}}$ 1

1

b) Gerade durch D, $\perp$ ABC-Ebene

$P_0 = D$, Richtungsvektor $\underline{v} = \underline{n}$ gut 1

$\begin{cases} x(t) = -1 + t \\ y(t) = 2 - 2t \\ z(t) = 2 - t \end{cases}$ 2p $t\in\mathbb{R}$ 1p

bzw $\quad x + 1 = \frac{y-2}{-2} = \frac{z-2}{-1}$