AlG1 | 3. PRÜFUNG | 9. JAN 2025.

(1) $u = \sqrt{2} + \sqrt{2}\cdot i$ und $v = 4\cdot\sqrt{2}i$

$u = 2\left(\cos\frac{\pi}{4} + i\cdot\sin\frac{\pi}{4}\right) \Rightarrow u^3 = 8\cdot\left(\cos\frac{3\pi}{4} + i\cdot\sin\frac{3\pi}{4}\right) =$

$= 8\cdot\left(-\frac{\sqrt{2}}{2} + i\cdot\frac{\sqrt{2}}{2}\right)$ (2p)

$u^3 + \bar{v} = -4\sqrt{2} + \cancel{4\sqrt{2}i} + (\cancel{-4\sqrt{2}i}) = -2^{5/2}$ (2p)

$\sqrt[4]{u^3+\bar{v}} = \sqrt[4]{-2^{5/2}} = 2^{5/8}\cdot\sqrt[4]{\cos\pi + i\cdot\sin\pi} =$ (1p)

$= 2^{5/8}\left(\cos\frac{\pi + k\cdot 2\pi}{4} + i\cdot\sin\frac{\pi + k 2\pi}{4}\right)$ $k = 0,1,2,3$

$z_1 = 2^{5/8}\cdot\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \sqrt{2}\cdot 2^{1/8}\cdot\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot i\right) = 2^{1/8}\cdot(1+i)$

$z_2 = 2^{5/8}\cdot\left(\cos\frac{3\pi}{4} + i\cdot\sin\frac{3\pi}{4}\right) = 2^{1/8}\cdot(-1+i)$

$z_3 = 2^{5/8}\cdot\left(\cos\frac{5\pi}{4} + i\cdot\sin\frac{5\pi}{4}\right) = 2^{1/8}\cdot(-1-i)$ (2p)

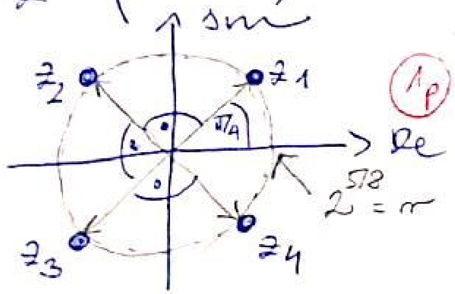
$z_4 = 2^{5/8}\cdot\left(\cos\frac{7\pi}{4} + i\cdot\sin\frac{7\pi}{4}\right) = 2^{1/8}\cdot(1-i)$

$z_1 = \sqrt[8]{2} + \sqrt[8]{2}\,i$ (1p)

$z_2 = -\sqrt[8]{2} + \sqrt[8]{2}\cdot i$

$z_3 = -\sqrt[8]{2} - \sqrt[8]{2}\cdot i$

$z_4 = \sqrt[8]{2} - \sqrt[8]{2}\cdot i$



(2)

a) $\lim\limits_{n\to\infty}\frac{n^{5/2} + n^{4/2}}{3\cdot n^{1/2} - n^{3/2}} = \lim\limits_{n\to\infty}\frac{n + n^{\frac{1}{2}}}{3\cdot\frac{1}{n} - 1} = -\infty$ (1p) (1p)

$\text{"}\frac{\infty}{-\infty}\text{"}$ (1p) $\quad \frac{1}{n}\to 0 \quad \text{"}\frac{\infty}{-1}\text{"}$ (1p)

(2) b) $\lim\limits_{x\to 0^+} x^3\cdot\ln(2x) = \lim\limits_{x\to 0^+}\frac{\ln 2x}{\frac{1}{x^3}} = 0$ (1p)

$\text{"}0\cdot(-\infty)\text{"}$ (1p) $\quad \text{"}\frac{-\infty}{+\infty}\text{"}$ $\quad$ L'Hosp. (1p)

$? \lim\limits_{x\to 0^+}\frac{\frac{1}{x}}{-3\cdot\frac{1}{x^4}} = \lim\limits_{x\to 0^+}\frac{-1}{3}\cdot x^3 = 0$ (1p)

(3) $f(x) = \frac{x}{e^{\sqrt{x}}}$ $\qquad D_f = \mathbb{R}_0^+$ (1p)

$f'(x) = \frac{e^{\sqrt{x}} - x\cdot e^{\sqrt{x}}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x}}}{(e^{\sqrt{x}})^2} = \frac{1 - \frac{\sqrt{x}}{2}}{e^{\sqrt{x}}} = 0 \Leftrightarrow x_1 = 4$ (1p)

	$]0;4[$	4	$x>4$
f'	+	0	—
f	echt mon. steig.	lok. max	echt mon. fallend

(2p) $\quad f(4) = f_{max} = \frac{4}{e^2}$

$f''(x) = \frac{-\frac{1}{4}\cdot\frac{1}{\sqrt{x}}\cdot e^{\sqrt{x}} - \left(1 - \frac{\sqrt{x}}{2}\right)\cdot e^{\sqrt{x}}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x}}}{(e^{\sqrt{x}})^2} =$

$= \underbrace{-\frac{1}{4}\cdot\frac{1}{\sqrt{x}}}_{(-)}\cdot\frac{1 + (2 - \sqrt{x})}{\underbrace{e^{\sqrt{x}}}_{(+)}} = 0 \Leftrightarrow \boxed{x_2 = 9}$ (2p)

	$]0;9[$	9	$x>9$
f''	—	0	+
f	konkav	IP	konvex

(2p)

4

$$\begin{cases} x(t) = 2\cdot(t-\sin t) \\ y(t) = 2\cdot(1-\cos t) \end{cases} \qquad t \in [0;2\pi]$$

$$\dot x(t) = 2(1-\cos t) \qquad \dot y(t) = 2\sin t$$

$$(\dot x(t))^2 = 4(1-2\cos t+\cos^2 t)\,;\ (\dot y(t))^2 = 4\cdot\sin^2(t)$$

$$A_M = 2\pi\cdot\int_{t=0}^{2\pi} y(t)\cdot\sqrt{(\dot x(t))^2+(\dot y(t))^2}\cdot dt =$$

$$=2\pi\cdot\int_{t=0}^{2\pi} 2(1-\cos(t))\cdot\sqrt{4\cdot(2-2\cos t)}\,dt = \quad (2p)$$

$$= 2\pi\cdot 4\cdot\sqrt{2}\cdot\int_{t=0}^{2\pi} 2\sin^2\left(\tfrac{t}{2}\right)\cdot\sqrt{2\cdot\sin^2\left(\tfrac{t}{2}\right)}\,dt =$$

$$= 32\pi\cdot\int_{t=0}^{2\pi}\left(1-\cos^2\left(\tfrac{t}{2}\right)\right)\cdot\sin\left(\tfrac{t}{2}\right)dt =$$

$$= 32\pi\cdot\int_0^{2\pi}\sin\left(\tfrac{t}{2}\right)dt + 32\pi\cdot\int_0^{2\pi}\cos^2\left(\tfrac{t}{2}\right)\cdot\left(-\sin\tfrac{t}{2}\right)dt =$$

$$= 32\pi\cdot\left[2\cdot\left(-\cos\tfrac{t}{2}\right)\right]_0^{2\pi} + 32\pi\cdot\left[\frac{\cos^3\left(\frac{t}{2}\right)}{\frac{3}{2}}\right]_0^{2\pi} = \quad (2p)$$

$$= 64\pi\cdot(+1-1) + \frac{64\pi}{3}\cdot(-1-1) = \left(128-\frac{128}{3}\right)\pi = \frac{256}{3}\pi \quad (2p)$$

$$V = \pi\cdot\int_{t=0}^{2\pi} y^2(t)\cdot\dot x(t)\,dt = \pi\cdot\int_0^{2\pi} 4(1-2\cos t+\cos^2 t)\cdot 2\cdot(1-\cos t)\,dt$$

$$= 8\pi\cdot\int_0^{2\pi} 1 \underbrace{-3\cos t}_{\int_0^{2\pi}\ldots=0} + \underbrace{3\cos^2 t}_{\frac{3}{2}(1+\cos(2t)),\ \int_0^{2\pi}\ldots=0} \underbrace{-\cos^3(t)}_{\int_0^{2\pi}\ldots=0}\,dt = 8\pi\cdot\int_0^{2\pi}\frac{5}{2}\,dt =$$

$$= 20\pi\cdot 2\pi =$$

$$= 40\pi^2 \quad (2p)$$

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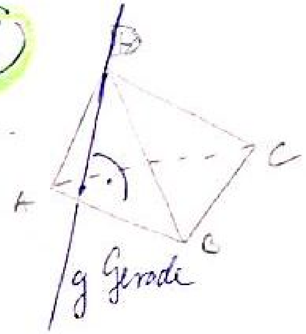

g ⊥ ABC-Ebene

$\Rightarrow \underline{v}_g = \underline{n}_{ABC}$ (1p)

$A(4;-1;2)$ $\qquad \vec{AB} = \begin{bmatrix}1\\3\\3\end{bmatrix} = \underline{a}$

$B(5;2;5)$ $\qquad \vec{AC} = \begin{bmatrix}-3\\0\\-1\end{bmatrix} = \underline{b}$ (1p)

$C(1;-1;1)$

$D(5;-2;1)$ $\qquad \vec{AD} = \begin{bmatrix}1\\-1\\-1\end{bmatrix} = \underline{c}$

$$\underline{abc} = \begin{vmatrix}1 & 3 & 3\\ -3 & 0 & -1\\ 1 & -1 & -1\end{vmatrix} = 1(0-1)-3\cdot(3+1)+3\cdot(3-0) = -4 \quad (1p)$$

$$\Rightarrow V_{Spat} = |-4| = 4 \ (1p) \Rightarrow V_{ABCD\,Tetrae} = \frac{V_{Spat}}{6} = \frac{4}{6} \quad (1p)$$

$$\vec{AB}\times\vec{AC} = \begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 1 & 3 & 3\\ -3 & 0 & -1\end{vmatrix} = \begin{bmatrix}+(-3-0)\\-(-1+9)\\+(0+9)\end{bmatrix} = \begin{bmatrix}-3\\-8\\9\end{bmatrix} = \underline{n}_{ABC\text{-}Ebene} \quad (1p)$$

$g:\ \underline{v}_g = \begin{bmatrix}-3\\-8\\9\end{bmatrix}$ ein Aufpunkt: $D(5;-2;1)$

$$g: \begin{cases} x(t) = 5-3t \\ y(t) = -2-8t \\ z(t) = 1+9t \end{cases} \qquad t\in\mathbb{R} \quad (2p)$$

Oder: $\dfrac{x-5}{-3} = \dfrac{y+2}{-8} = \dfrac{z-1}{9}$