TESTAUFGABE 1. [illegible] 2024.

① $\begin{vmatrix} -2^{+} & 1^{-} & p^{+} \\ 1 & 1 & 1 \\ 4 & -1 & 2 \end{vmatrix} = -2\cdot(2+1) - 1\cdot(2-4) + p\cdot(-1-4) \neq 0$ 1p 1p 1p

$-4-5p \neq 0$

$\underline{u}, \underline{v}, \underline{w}$ lin. unabh. $\Longleftrightarrow p \neq -\frac{4}{5}$ 1p

② $\alpha\cdot\begin{bmatrix}4\\5\\2\end{bmatrix} + \beta\cdot\begin{bmatrix}1\\-1\\1\end{bmatrix} + \gamma\begin{bmatrix}0\\2\\-1\end{bmatrix} = \begin{bmatrix}-3\\0\\-4\end{bmatrix}$ 1p

$\Updownarrow$

$\begin{cases} 4\alpha + \beta = -3 \\ 5\alpha - \beta + 2\gamma = 0 \\ 2\alpha + \beta - \gamma = -4 \end{cases}$ 1p

$\begin{cases} 9\alpha + 2\gamma = -3 \\ -2\alpha - \gamma = -1 \end{cases}$ $\gamma = 1-2\alpha$

$9\alpha + 2(1-2\alpha) = -3$

$5\alpha = -5$ $\boxed{\alpha = -1}$

$\beta = -3 - 4\alpha = -3 + 4 = 1$ 2p $\boxed{\gamma = 3}$

$\underline{v} = -1\cdot\underline{a} + 1\cdot\underline{b} + 3\cdot\underline{c}$ 1p

③ $x^3 + \sqrt{3}\cdot i - 1 = 0$

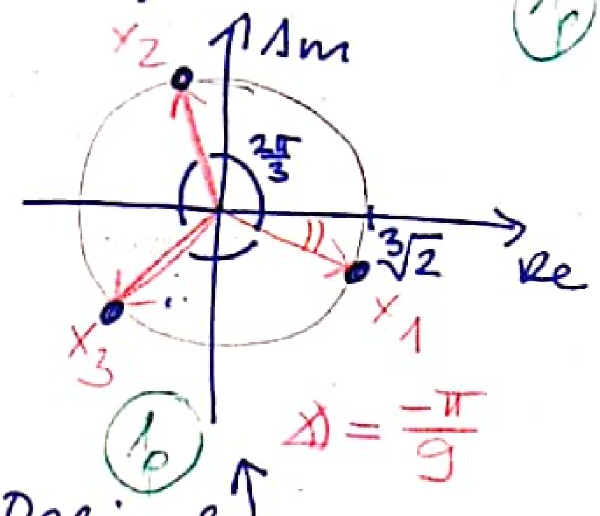
$x^3 = 2\cdot\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\cdot i\right) = 2\cdot\left(\cos\frac{-\pi}{3} + i\cdot\sin\frac{-\pi}{3}\right)$ 1p

$x = \sqrt[3]{2}\cdot\left(\cos\frac{-\frac{\pi}{3}+k2\pi}{3} + i\cdot\sin\frac{-\frac{\pi}{3}+k2\pi}{3}\right)$ $k = 0, 1, 2$ 1p

$x_1 = \sqrt[3]{2}\cdot\left(\cos\frac{-\pi}{9} + i\cdot\sin\frac{-\pi}{9}\right)$

$x_2 = \sqrt[3]{2}\cdot\left(\cos\frac{5\pi}{9} + i\cdot\sin\frac{5\pi}{9}\right)$ 1p

$x_3 = \sqrt[3]{2}\cdot\left(\cos\frac{11\pi}{9} + i\cdot\sin\frac{11\pi}{9}\right)$



↳ bilden ein regelmäßiges Dreieck

4) $P(x) = x^4 - 5x^3 + 10x^2 - 10x + 4$

Kandidaten $\pm 1, \pm 2, \pm 4$

$x_1 = 1$ Wurzel ✓ 1p

$$x^4 - 5x^3 + 10x^2 - 10x + 4 : (x-1) = x^3 - 4x^2 + 6x - 4$$

$$\begin{array}{l} x^4 - x^3 \\ \hline -4x^3 + 10x^2 \\ -4x^3 + 4x^2 \\ \hline 6x^2 - 10x \\ 6x^2 - 6x \\ \hline -4x + 4 \\ -4x + 4 \\ \hline 0 \end{array}$$

1p

$x_2 = 2$ Wurzel ✓ 1p

$$x^3 - 4x^2 + 6x - 4 : (x-2) = x^2 - 2x + 2$$

$$\begin{array}{l} x^3 - 2x^2 \\ \hline -2x^2 + 6x \\ -2x^2 + 4x \\ \hline 2x - 4 \\ 2x - 4 \\ \hline 0 \end{array}$$

$$x_{3,4} = \frac{2 \pm \sqrt{4-8}}{2} \begin{cases} 1-i \\ 1+i \end{cases}$$

1p

über $\mathbb{R}$: $(x-1)(x-2)(x^2-2x+2) = P(x)$

über $\mathbb{C}$: $(x-1)(x-2)(x-(1-i))(x-(1+i)) = P(x)$

2 1p

5) $A \in \mathbb{R}$ ist r Grenzwert r reellen Zahlenfolge $a_n \Leftrightarrow \forall \varepsilon > 0\ \exists \nu \in \mathbb{N}$: wenn $n > \nu$, dann $|a_n - A| < \varepsilon$.

2p

CamScannerrel szkennelve