A262 | 2. TESTAUFG. | 2025.

(1) $\underbrace{6xy+8y^2}-9=0$

$$\underline{\underline{A}}=\begin{bmatrix}0&3\\3&8\end{bmatrix} \Rightarrow \begin{vmatrix}-\lambda&3\\3&8-\lambda\end{vmatrix}=\lambda^2-8\lambda-9=0$$

$$\lambda_1=-1,\ \lambda_2=9 \Leftarrow (\lambda+1)(\lambda-9)=0 \quad 1$$

$\lambda_1=-1$ $\begin{bmatrix}+1&3\\3&9\end{bmatrix}\sim\begin{bmatrix}1&3\\0&0\end{bmatrix}$ $\underline{s}_{10}=\begin{bmatrix}3\\-1\end{bmatrix}\cdot\frac{1}{\sqrt{10}}$

$\lambda_2=9$ $\begin{bmatrix}-9&3\\3&-1\end{bmatrix}\sim\begin{bmatrix}3&-1\\0&0\end{bmatrix}$ $\underline{s}_{20}=\begin{bmatrix}1\\3\end{bmatrix}\cdot\frac{1}{\sqrt{10}}$

$[s_{10}\ s_{20}]=\underline{\underline{B}}^{-1}=\underline{\underline{B}}^{T}$

$i, j \leftrightarrow s_1, s_2$

$\underline{\underline{B}}=(\underline{\underline{B}}^{-1})^{-1}=(\underline{\underline{B}}^{-1})^{T}$

$\underline{\underline{A}}_{diag}=\begin{bmatrix}-1&0\\0&9\end{bmatrix}$

Basiswechsel...

$$[x\,y]\cdot\underline{\underline{B}}^T\cdot\underline{\underline{B}}\cdot\underline{\underline{A}}\cdot\underline{\underline{B}}^T\cdot\underline{\underline{B}}\cdot\begin{bmatrix}x\\y\end{bmatrix}-9=0$$

$$[\eta\ \xi]\cdot\underline{\underline{A}}_{diag}\cdot\begin{bmatrix}\eta\\\xi\end{bmatrix}-9=0 \quad 1$$

$$-\eta^2+9\xi^2-9=0$$

$$-\frac{\eta^2}{3^2}+\frac{\xi^2}{1^2}=1 \quad 1 \quad \text{Hyperbel!} \quad 1$$

(2) $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}\cdot(n+1)}{2^n}$ #

Abs. konv.? $\sum_{n=0}^{\infty}\frac{n+1}{2^n}$ (*) $\quad \frac{|a_{n+1}|}{|a_n|}=\frac{n+2}{2^{n+1}}\cdot\frac{2^n}{n+1}\xrightarrow[\infty]{n}\frac{1}{2}<1$

(*) konv. $\Leftarrow$ W.Kr.

abs. konv. $\Leftarrow$ Def. 2p

Leibnitz? alternierend ✓, $a_n \to 0$ ✓

Monotonie? $|a_{n+1}| \leq |a_n|$

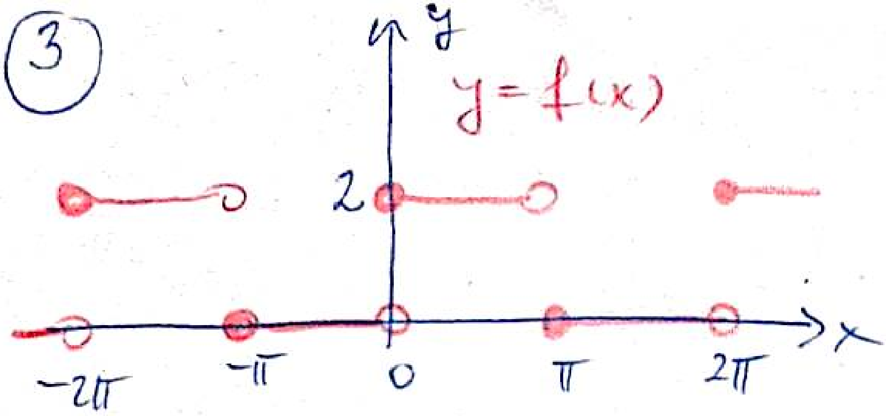
$\Updownarrow$ $\frac{n+2}{2^{n+1}} \leq \frac{n+1}{2^n}$ $\quad |a_n| \downarrow 0$ ✓

$\Updownarrow$ $n+2 \leq 2n+2$ 3p

Also: # Leibnitz-Reihe ⇒ konvergent UND # ist auch absolut konv.

CamScannerrel szkennelve

3



$g(x) := f(x) - 1$

schon UNGERADE

$G(x) = \sum_{k=1}^{\infty} b_k \cdot \sin(kx)$

und $F(x) = G(x) + 1$

$$b_k = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} g(x) \cdot \sin(kx)\,dx = \frac{2}{\pi} \cdot \int_{0}^{\pi} 1 \cdot \sin(kx)\,dx =$$

$$= \frac{2}{\pi} \cdot \left[\frac{-\cos(kx)}{k} \right]_0^{\pi} = \frac{-2}{k\pi} \cdot \left((-1)^k - 1 \right) = \begin{cases} \frac{4}{k\pi} & k \text{ ung.} \\ 0 & k \text{ gerade} \end{cases}$$

2

$$G(x) = \sum_{m=1}^{\infty} \frac{4}{(2m-1)\pi} \cdot \sin((2m-1)x)$$

$$F(x) = 1 + \sum_{m=1}^{\infty} \frac{4}{(2m-1)\pi} \cdot \sin((2m-1)x) =$$

$$= 1 + \frac{4}{\pi} \cdot \sin x + \frac{4}{3\pi} \sin(3x) + \frac{4}{5\pi} \cdot \sin(5x) + \ldots$$

1p

$F(x) = f(x)$ wenn $x \neq n \cdot \pi$ $n \in \mathbb{Z}$ 1p

$F(5\pi) = \frac{2+0}{2} = 1$ (Satz von Lipót Fejér) 1p

4

$$\int_{x=0}^{2} \cos(2x^3)\,dx = \int_0^2 \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \cdot (2x^3)^{2k} =$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k \cdot 4^k}{(2k)!} \cdot \int_0^2 x^{6k}\,dx = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot 4^k}{(2k)!} \cdot \left[\frac{x^{6k+1}}{6k+1} \right]_0^2 =$$

1

$$= \sum_{k=0}^{\infty} \frac{(-1)^k \cdot 4^k \cdot 2^{6k+1}}{(2k!) \cdot (6k+1)} = \sum_{k=0}^{\infty} \frac{(-1)^k \cdot 2^{8k+1}}{(2k!)(6k+1)} \approx$$

1

$$\approx 2 - \frac{2^9}{2 \cdot 7} + \frac{2^{17}}{24 \cdot 13} = 2 - \frac{2^8}{7} + \frac{2^{14}}{39}$$

1

$$|F| \leq \left| \frac{-2^{25}}{720 \cdot 19} \right| = \frac{2^{25}}{720 \cdot 19}$$

(Leibnitz-Reihe!) 1

+3p 2) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1} \cdot (n+1)}{2^n} = f\left(\frac{1}{2}\right)$.

$$\sum_{n=0}^{\infty} (-1)^{n+1} \cdot (n+1) \cdot x^n = f(x)$$

$$\int_{t=0}^{x} f(t)\,dt = \sum_{n=0}^{\infty} (-1)^{n+1} \cdot \left[t^{n+1}\right]_0^x = \sum_{0}^{\infty} (-1)^{n+1} \cdot x^{n+1} =$$

$$= -x \cdot \frac{1}{1-(-x)}$$

$$= \frac{-x}{1+x}$$

$$\left(\ \right)'_x$$

$$f(x) = \left(\frac{-x}{x+1}\right)' = \frac{-(x+1)+x\cdot 1}{(x+1)^2} = \frac{-1}{(x+1)^2}$$

$$f\left(\frac{1}{2}\right) = \frac{-1}{\left(\frac{3}{2}\right)^2} = \frac{-4}{9}$$