A262 | 3. PRÜFUNG | 2025.

(1) $\begin{bmatrix} -4 & 1 & 2 & | & 1 \\ 1 & -1 & 0 & | & 2 \\ 2 & 1 & c & | & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & | & 2 \\ 0 & -3 & 2 & | & 9 \\ 0 & 3 & c & | & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & | & 2 \\ 0 & -3 & 2 & | & 9 \\ 0 & 0 & c+2 & | & 8 \end{bmatrix}$ 2

$c=-2 \Leftrightarrow r(\underline{\underline{A}})=2 < r(\underline{\underline{A}}|\underline{b})=3 \Rightarrow$ keine Lösung 1

$c \neq -2 \Leftrightarrow r(\underline{\underline{A}}) = r(\underline{\underline{A}}|\underline{b}) = 3 = N \Rightarrow$ eindeutige Lösung. 1

Parametrische Lösung: kann nicht vorkommen. 1

$\boxed{c=0} \Rightarrow \det \underline{\underline{A}} = \begin{vmatrix} -4 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 1 & 0 \end{vmatrix} = 2(1+2) = 6 = D$ 1

$D_z = \begin{vmatrix} -4 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & 3 \end{vmatrix} = 1\overset{3}{(1+2)} - 2\overset{-6}{(-4-2)} + 3\overset{3}{(4-1)} = 24$ 1

$z = \frac{D_z}{D} = \frac{24}{6} = 4$ // 1

(2) $\mathcal{A}: \underline{i} \mapsto \underline{i},\ \underline{j} \mapsto \underline{j},\ \underline{k} \mapsto -\underline{k} \Rightarrow \underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ 2

$\mathcal{B}: \underline{i} \mapsto \underline{i},\ \underline{j} \mapsto -\underline{j},\ \underline{k} \mapsto \underline{k} \Rightarrow \underline{\underline{B}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 2

Komposition: $\underline{\underline{K}} = \underline{\underline{B}} \cdot \underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ 2

$\lambda_1 = 1,\ \lambda_2 = \lambda_3 = -1,\ \underbrace{\underline{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}},\ \underbrace{\underline{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \wedge \underline{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$ z.B.

Invar. Unterräume: e x-Achse $\wedge$ die yz-Ebene 2

(3) $\sum_{n=1}^{\infty} n \cdot x^n = x \cdot \sum_{n=1}^{\infty} n \cdot x^{n-1} = f(x) = x \cdot g(x)$ 1

$\int_{t=0}^{x} g(t)\,dt = \sum_{n=1}^{\infty} \underbrace{\int_{t=0}^{x} n \cdot t^{n-1}\,dt}_{[t^n]_0^x} = \sum_{n=1}^{\infty} x^n = \frac{x}{1-x}$ 1

$$g(x) = \left(\frac{x}{1-x}\right)' = \frac{1(1-x) - x\cdot(-1)}{(1-x)^2} = \frac{1-x+x}{(1-x)^2} = \frac{1}{(1-x)^2}$$ 1

$$\sum_{n=1}^{\infty} n\cdot x^n = x\cdot g(x) = \frac{x}{(1-x)^2}$$ 1

KR: $|x| < 1 \Rightarrow KR = 1$ 1

KB: $\left.\begin{array}{l} x=-1 \Rightarrow \sum n\cdot(-1)^n \text{ div.} \\ x = 1 \Rightarrow \sum n \cdot 1^n \text{ div.} \end{array}\right\} \Rightarrow KB = ]-1;1[$ 1

$$x = \frac{-1}{4} \Rightarrow \sum_{n=1}^{\infty} n\cdot\left(\frac{-1}{4}\right)^n = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{n}{4^n} = -\frac{1}{4} + \frac{2}{16} - \frac{3}{64} + - \dots$$

ALSO: $\frac{-1}{4} + \frac{2}{16} - \frac{3}{64} + - \dots + (-1)^n \cdot \frac{n}{4^n} + \dots = \frac{-\frac{1}{4}}{\left(1+\frac{1}{4}\right)^2} =$

$$= \frac{-1}{4}\cdot\frac{16}{25} = \frac{-4}{25}$$ 2

4 $z = \frac{x+2}{y^2+1}$ Niveaulinien: $z \equiv c \; (c \in \mathbb{R})$ 1

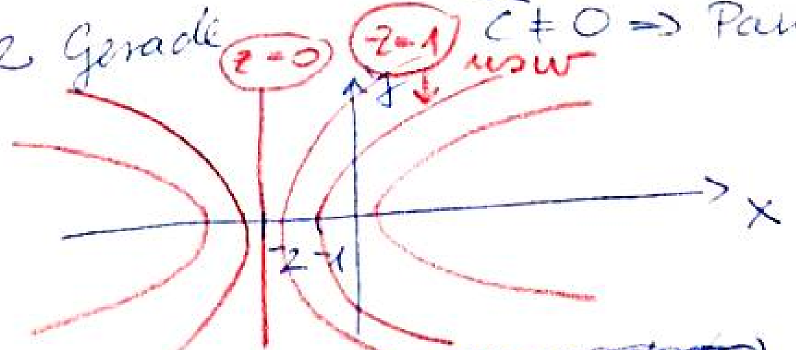
$$\frac{x+2}{y^2+1} = c$$

$$x+2 = c\cdot y^2 + c$$

$$x = c\cdot y^2 + (c-2)$$ 1

$c = 0 \Rightarrow x = -2$ Gerade $c \neq 0 \Rightarrow$ Parabeln

2




1

$f'_x(x,y) = \frac{1}{y^2+1}$ ; $f'_y(x,y) = \frac{(x+2)\cdot(-1)\cdot 2y}{(y^2+1)^2}$ ; $P(-1;2)$ 1

Tang. ebene:

$$z = \frac{1}{5}(x+1) + \frac{-4}{25}(y-2) + \frac{1}{5}$$ 2

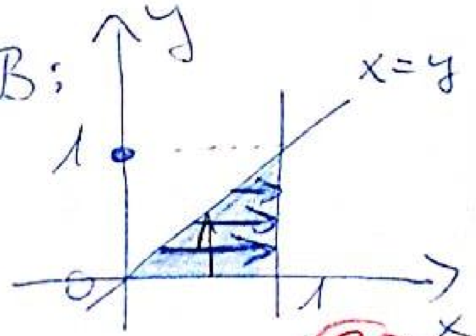
CamScannerrel szkennelve

(5)

$$I=\int_{y=0}^{1}\int_{x=y}^{1} y^2\cdot \underbrace{e^{-x^4}}_{\text{KEINE elementare Stammfunktion!}}\,dx\,dy$$

1

B:




2P

$$I=\int_{x=0}^{1}\int_{y=0}^{x} y^2\cdot \boxed{e^{-x^4}}\,dy\,dx =$$

2

$$=\int_{x=0}^{1} e^{-x^4}\cdot\left[\frac{y^3}{3}\right]_0^x dx=\frac{1}{3}\cdot\int_{x=0}^{1} e^{-x^4}\cdot x^3\,dx=$$

1

$$=\frac{-1}{12}\cdot\int_{x=0}^{1} e^{-x^4}\cdot(-4x^3)\,dx=\frac{-1}{12}\cdot\left[e^{-x^4}\right]_0^1=$$

1

$$=\frac{-1}{12}\cdot\left(e^{-1}-e^{0}\right)=\frac{1-\frac{1}{e}}{12}=\frac{e-1}{12e}$$

1