ALG2 (4. PRÜFUNG) 2025.

(1)
$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 2 & -1-\lambda & 0 \\ -1 & -3 & 2-\lambda \end{vmatrix} = (1-\lambda)(-1-\lambda)(2-\lambda) = 0 \iff$$

2

$$\lambda_1 = -1,\ \lambda_2 = 1,\ \lambda_3 = 2$$

$$\lambda_1 = -1 \Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 0 \\ -1 & -3 & 3 \end{bmatrix} \sim \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \underline{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \text{ gut}$$

$$\lambda_2 = 1 \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 2 & -2 & 0 \\ -1 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & -4 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \underline{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \text{ gut}$$

2

$$\lambda_3 = 2 \Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ -1 & -3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \underline{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ gut}$$

$$\underline{\underline{B}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 1 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & -4 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -3 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{\begin{matrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ -3 & -1 & 1 \end{matrix}}_{\underline{\underline{B}}^{-1}} \\ 0 & 0 & 1 & \end{array}\right]$$

2

$$\begin{tikzcd} \boxed{B_{NAT}} \arrow[r, "\underline{\underline{A}}"] \arrow[d, bend right, "\underline{\underline{B}}^{-1}"'] & \boxed{} \arrow[d, bend right] \\ \{v_i\} \arrow[u, bend right, "\underline{\underline{B}}"'] \arrow[r, "\underline{\underline{A}}_d"] & \bigcirc \arrow[u, bend right] \end{tikzcd}$$

$$\underline{\underline{A}}_d = \underline{\underline{B}}^{-1} \cdot \underline{\underline{A}} \cdot \underline{\underline{B}} =$$

$$= \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ -3 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ -6 & -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

✓

2

(2) $\sum_{k=1}^{\infty}(-1)^k \cdot \frac{5}{3^k+1}$ (*)

$$\sum_{k=1}^{\infty}|a_k| = \underbrace{\sum_{k=1}^{\infty}\frac{5}{3^k+1}}_{} < \underbrace{\sum_{k=1}^{\infty}\frac{5}{3^k} = 5\cdot\sum_{k=1}^{\infty}\left(\frac{1}{3}\right)^k \quad \left|\frac{1}{3}\right|<1.}_{\text{konv. pos. geom. num. Reihe}}$$ (2) (1)

konv. $\xLeftarrow{\text{V-Krit}}$ (1)

$\Downarrow$

(*) ABS. konvergent ✓

$$\underbrace{\sum_{k=1}^{\infty}(-1)^k\cdot\frac{5}{3^k+1}}_{\text{Leibnitz-Reihe}} \approx \frac{-5}{4}+\frac{5}{10}-\frac{5}{28}+\dots+(-1)^N\cdot\frac{5}{3^N+1}$$ (1)

$\Downarrow$

$$F_N < |a_{N+1}| = \frac{5}{3^{N+1}+1} < \frac{1}{100} \quad \Downarrow$$

$\boxed{3, 9, 27, 81, 243, 729 \dots}$ $\quad 499 < 3^{N+1}$

$N+1 = 6$ schon gut (2)

$\boxed{N=5}$

(*) $\approx \frac{-5}{4}+\frac{5}{10}-\frac{5}{28}+\frac{5}{82}-\frac{5}{244}$ (1)

(3) $\sum_{n=1}^{\infty}\frac{x^n}{n} = f(x)$

$$\left|\frac{x^{n+1}}{n+1}\right|\cdot\left|\frac{n}{x^n}\right| < 1$$

$$|x|\cdot\underbrace{\frac{n}{n+1}}_{\to 1} < 1$$

$x=-1 \quad \sum(-1)^n\cdot\frac{1}{n}$ KONV $\Leftarrow \boxed{|x|<1}$ (2p)

$x=1 \quad \sum\frac{1}{n}$ DIV. Also: KB $= [-1; 1[$

$$f'(x) = \sum_{n=1}^{\infty}x^{n-1} = \sum_{n=0}^{\infty}x^n = \frac{1}{1-x}$$ (2)

$$f(x) = \int_{t=0}^{x}\frac{1}{1-t}\,dt = \Big[-\ln|1-t|\Big]_0^x = -\ln|1-x| = -\ln(1-x)$$ auf KB. (2)

$$\Rightarrow \sum_{n=1}^{\infty}\frac{\left(-\frac{1}{3}\right)^n}{n} = -\ln\left(1+\frac{1}{3}\right) = -\ln\frac{4}{3} = \ln\frac{3}{4}$$ (2)

$-\frac{1}{3}\in$ KB

④ $f(x;y) = y^2 \cdot \ln(x^2+1) - \frac{xy+y^2}{x+1}$ $\quad P(0;2)$

$f'_x(x;y) = y^2 \cdot \frac{2x}{x^2+1} - \frac{y(x+1)-(xy+y^2)}{(x+1)^2}$ 1 $\quad \underline{v}(2;3)$

$f'_y(x;y) = 2y \cdot \ln(x^2+1) - \frac{x+2y}{x+1}$ 1 $\quad \Downarrow \quad \underline{v}_0 = \frac{1}{\sqrt{13}} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$f'_x(0;2) = 2 \quad f'_y(0;2) = -4 \quad f(0;2) = -4$ 1

Gl. v Tang.ebene: $z = 2(+x-0) - 4(y-2) - 4$ 2

$0 = 2x - 4y - z + 4$

Einheitsflächennormale: $\underline{n}_0 = \frac{1}{\sqrt{21}} \cdot \begin{bmatrix} 2 \\ -4 \\ -1 \end{bmatrix}$ 1

$\frac{\partial f}{\partial \underline{v}}(P) = \left\langle \begin{bmatrix} 2 \\ -4 \end{bmatrix}; \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \frac{1}{\sqrt{13}} \right\rangle = \frac{1}{\sqrt{13}} \cdot (-8) = \frac{-8}{\sqrt{13}}$ 1 1

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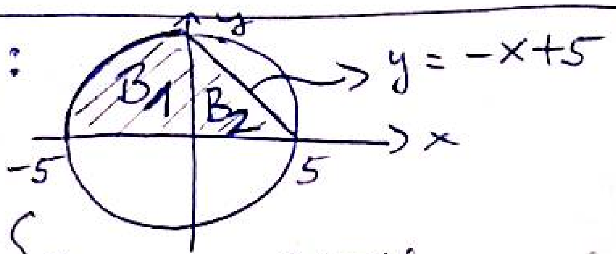
⑤ $\mu(x;y) = y$ $\qquad$ B:



$M = \iint_B y \, dF$

$M_1 = \iint_{B_1} y \, dF = \int_{\varphi=\frac{\pi}{2}}^{\pi} \int_{r=0}^{5} r^2 \cdot \sin\varphi \, dr \, d\varphi$

B_1: $x = r \cdot \cos\varphi$, $y = r \cdot \sin\varphi$, $|J| = r$, $r \in [0,5]$, $\varphi \in [\frac{\pi}{2}; \pi]$

$= [-\cos\varphi]_{\pi/2}^{\pi} \cdot \left[\frac{r^3}{3}\right]_0^5 = (1+0) \cdot \frac{125}{3}$ 3

$M_2 = \iint_{B_2} y \, dF = \int_{x=0}^{5} \int_{y=0}^{-x+5} y \, dy \, dx = \int_{x=0}^{5} \frac{1}{2} \cdot [y^2]_0^{-x+5} dx =$

$= \frac{1}{2} \cdot \left[\frac{x^3}{3} - 5x^2 + 25x\right]_0^5 =$

$= \frac{1}{2}\left(\frac{125}{3} - 125 + 125\right) = \frac{125}{6}$ 3

$M = \frac{125}{3} + \frac{125}{6} = \frac{125}{2} = 62{,}5$ (M.einh.) 2