A2G2 TESTA. 1. 2025.

(1) $\underline{\underline{A}} = \begin{bmatrix} 9 & 1 \\ 4 & 1 \end{bmatrix}$ $\underline{\underline{B}} = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ $\underline{\underline{C}} = \begin{bmatrix} 3 & +1 & 1 \\ 4 & -1 & 1 \end{bmatrix}$

$\det \underline{\underline{A}} = 9 - 4 = 5$ $\quad \operatorname{adj} \underline{\underline{A}} = \begin{bmatrix} +1 & -4 \\ -1 & +9 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 \\ -4 & 9 \end{bmatrix}$ 2p

$\underline{\underline{A}}\,\underline{\underline{X}} + \underline{\underline{B}} = \underline{\underline{C}} \qquad \underline{\underline{X}} = \underline{\underline{A}}^{-1} \cdot (\underline{\underline{C}} - \underline{\underline{B}})$

$\underline{\underline{X}} = \frac{1}{5} \cdot \begin{bmatrix} 1 & -1 \\ -4 & 9 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & 2 \\ 4 & -2 & -1 \end{bmatrix} = \frac{1}{5} \cdot \begin{bmatrix} 0 & 2 & 3 \\ 20 & -18 & -17 \end{bmatrix}$

1p

$= \begin{bmatrix} 0 & 2/5 & 3/5 \\ 4 & -18/5 & -17/5 \end{bmatrix}$ 1p

(2) $\left[\begin{array}{cccc|c} 3 & -1 & 4 & c & 2 \\ -1 & -1 & 2 & 0 & -1 \\ 0 & 2 & 1 & -1 & 0 \\ 1 & 1 & 1 & 2 & 5 \end{array}\right] \underset{\substack{z_4 + z_2 \\ z_2 \cdot (-1)}}{\overset{z_1 + 3z_2}{\sim}} \left[\begin{array}{cccc|c} 0 & -4 & 10 & c & -1 \\ 1 & 1 & -2 & 0 & 1 \\ 0 & 2 & 1 & -1 & 0 \\ 0 & 0 & 3 & 2 & 4 \end{array}\right] \underset{\text{Zeilen-vertausch}}{\overset{z_1 + z_3 \cdot 2}{\sim}}$

$\sim \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 1 \\ 0 & 2 & 1 & -1 & 0 \\ 0 & 0 & 12 & c-2 & -1 \\ 0 & 0 & 3 & 2 & 4 \end{array}\right] \underset{z_3 \leftrightarrow z_4}{\overset{z_3 - 4z_4}{\sim}} \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 1 \\ 0 & 2 & 1 & -1 & 0 \\ 0 & 0 & 3 & 2 & 4 \\ 0 & 0 & 0 & c-10 & -17 \end{array}\right]$ 2p

$c = 10 \Rightarrow r(\underline{\underline{A}}) = 3 < r(\underline{\underline{A}} \mid \underline{b}) = 4$ LGLSyst nicht lösbar 1p

$c \neq 10 \Rightarrow r(\underline{\underline{A}}) = r(\underline{\underline{A}} \mid \underline{b}) = n = 4$ LGLS hat eine eindeutige Lösung 1p

$\boxed{2B}$ $c = 44 \Rightarrow u = \frac{-17}{34} = \frac{-1}{2} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 2 & 1 & -\frac{1}{2} \\ 0 & 0 & 3 & 5 \end{array}\right] \sim$

$z = \frac{5}{3}$

$y = \frac{-13}{12}$

$x = \frac{65}{12}$

$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 30/6 \\ 0 & 2 & 0 & -\frac{13}{6} \\ 0 & 0 & 1 & \frac{5}{3} \end{array}\right]$

1p

(3) $\underline{\underline{M}} = \begin{bmatrix} 7 & 3 & 0 \\ -2 & 0 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ $\quad \begin{vmatrix} 7-\lambda & 3 & 0 \\ -2 & -\lambda & 0 \\ 4 & 2 & 1-\lambda \end{vmatrix} =$

$= (1-\lambda) \cdot \underbrace{\left[\lambda^2 - 7\lambda + 6\right]}_{(\lambda-1)(\lambda-6)} = 0$ 2p

$\lambda_1 = \lambda_2 = 1$
alg. Multipl. = zwei

$\lambda_3 = 6$ alg. Mult. = eins

$\underline{s}_1, \underline{s}_2$? $\quad \begin{bmatrix} 6 & 3 & 0 \\ -2 & -1 & 0 \\ 4 & 2 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad 2x + y = 0$

2p

$\underline{s}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot t \qquad \underline{s}_2 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} \cdot \tau \qquad t, \tau \in \mathbb{R} \setminus \{0\}$

geom. Mult. von $\lambda = 1$ ist zwei

$\underline{s}_3$? $\quad \begin{bmatrix} 1 & 3 & 0 \\ -2 & -6 & 0 \\ 4 & 2 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 \\ 0 & -10 & -5 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \left.\begin{array}{l} x + 3y = 0 \\ 2y + z = 0 \end{array}\right\}$

$\underline{s}_3 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \cdot \Theta \qquad \Theta \in \mathbb{R} \setminus \{0\}$ 1p

geom. Mult. von $\lambda = 6$ ist eins.

Multipl.-en: 1p

(4) $\underline{\underline{R}}_{x, \frac{\pi}{4}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$ 1p $\quad \left\{ \begin{array}{l} \underline{i} \longmapsto \underline{i} \\ \underline{j} \longmapsto \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \\ \underline{k} \longmapsto \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \end{array} \right.$

$\underline{\underline{S}}_{xz} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 1p $\quad \underline{i} \longmapsto \underline{i} \quad \underline{j} \longmapsto -\underline{j} \quad \underline{k} \longmapsto \underline{k}$

Zuerst rotieren, dann drehen:

$\underline{\underline{K}} = \underline{\underline{S}} \cdot \underline{\underline{R}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$ 1p

1p