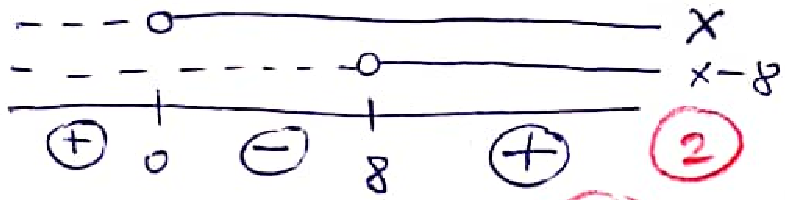
1. VIZSGA   KKA GIK   2024. 05. 28.

(1) $\sqrt{x+1} < \frac{x}{2} - 1$ Alaph.: $x \geq -1$ és $\frac{x}{2} - 1 \geq 0$

$x + 1 < \frac{x^2}{4} - x + 1$ $\boxed{x \geq 2}$ (2) $x \geq 2$

$0 < \frac{x^2}{4} - 2x$ (2)

$0 < \frac{1}{4}x \cdot (x-8)$



+ alaphalmaz $\Rightarrow$ Megoldás: $x \geq 2$ (2)

(2) $x^4 - x^2 - 12 = P(x)$ $P(x) = 0 \quad (x^2)_{1,2} = \frac{1 \pm \sqrt{1+48}}{2}$ ($\sqrt{1+48} = 7$)

$x^2_{(1)} = -3 \quad x^2_{(2)} = 4$ (2)

$x^2 = 4 \Leftrightarrow x_1 = -2$
$x_2 = 2$ (2)

$x^4 + 0x^3 - x^2 + 0x - 12 : (x^2 - 4) = x^2 + 3$ (2)
$x^4 + 0x^3 - 4x^2$
$3x^2 + 0x - 12$
$3x^2 + 0x - 12$
0

nincs valós gyöke

(2) $\Rightarrow P(x) = (x+2)(x-2)(x^2+3)$

(3) a) $\lim\limits_{n\to\infty} \frac{\sqrt{4n-1} - \sqrt{2n+1}}{\sqrt{n}} = \lim\limits_{n\to\infty} \frac{\sqrt{4n-1} - \sqrt{2n+1}}{\sqrt{n}} \cdot \frac{\sqrt{4n-1} + \sqrt{2n+1}}{\sqrt{4n-1} + \sqrt{2n+1}}$ (1)

„$\frac{\infty - \infty}{\infty}$" nem def.

$= \lim\limits_{n\to\infty} \frac{4n - 1 - 2n - 1}{\sqrt{n}(\sqrt{4n-1} + \sqrt{2n+1})} = \lim\limits_{n\to\infty} \frac{2n - 2}{\sqrt{4n^2 - n} + \sqrt{2n^2 + n}} =$ (1)

$= \lim\limits_{n\to\infty} \frac{2 - \frac{2}{n}}{\sqrt{4 - \frac{1}{n}} + \sqrt{2 + \frac{1}{n}}} = \frac{2}{2 + \sqrt{2}}$ (1) (1)

b) $\lim\limits_{x\to 0} \frac{\sin 2x}{4x} = \frac{1}{2} \cdot \lim\limits_{x\to 0} \frac{\sin 2x}{2x} = \frac{1}{2} \cdot 1 = \frac{1}{2}$ (1) (1)

„0/0"

4. $f(x)=e^{-x^2}$ 1) $D_f=\mathbb{R}$ 2) zh nincs
3) PÁROS fgv., nem period. 1

4) $\lim\limits_{x\to\pm\infty} e^{-x^2}=0 \Rightarrow$ csak vízszintes aszimptota $\pm\infty$-ben 1

5) $f'(x)=-2x\cdot e^{-x^2}$

$f(0)=f_{max}=1$

	$x<0$	$x=0$	$x>0$
f'	+	0	−
f	szig. m. nő	LOK MAX	szig. mon. csökk.

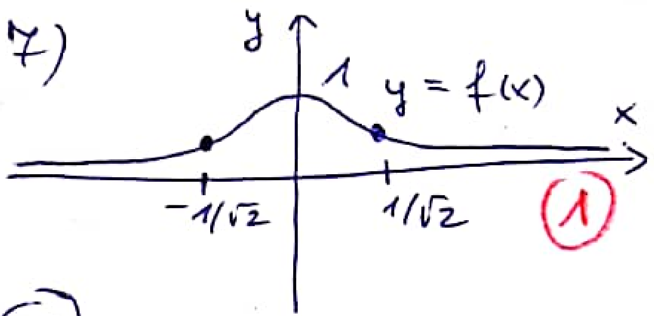
2

6) $f''(x)=(-2+(2x)^2)e^{-x^2}=2(2x^2-1)\cdot e^{-x^2}$

	$x<\frac{-1}{\sqrt2}$	$\frac{-1}{\sqrt2}$	$]\frac{-1}{\sqrt2};\frac{1}{\sqrt2}[$	$\frac{1}{\sqrt2}$	$x>\frac{1}{\sqrt2}$
f''	+	0	−	0	+
f	konvex	IP	konkáv	IP	konvex

2

7)



1

8) $R_f=]0;1]$ 1

5 a) $\int x\cdot\sin(x^2)\,dx=\frac{1}{2}\cdot\int \underbrace{2x\cdot\sin(x^2)}_{g'(x)\cdot f(g(x))}dx=\frac{1}{2}\cdot(-\cos x^2)+c$ 1

2

$c\in\mathbb{R}$ 1

b) $\int \underbrace{x}_{f}\cdot\underbrace{\cos x}_{g'}\,dx = x\cdot\sin x-\int \sin x\,dx=$ 1 1

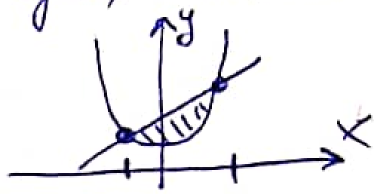
$f'=1,\ g=\sin x$ $\qquad =x\cdot\sin x+\cos x+c$ 1 $\quad c\in\mathbb{R}$ 1

6 $f(x)=2x^2+1$
$g(x)=7x+5$

$2x^2+1=7x+5$
$2x^2-7x-4=0$

$x_{1,2}=\frac{7\pm\sqrt{49+32}}{4}$ (9)

$x_1=\frac{-1}{2}$ $\quad x_2=4$ 2



2

$T=\int\limits_{x=-1/2}^{4} 7x+5-(2x^2+1)\,dx=\int\limits_{x=-1/2}^{4} -2x^2+7x+4\,dx=\left[\frac{-2x^3}{3}+\frac{7x^2}{2}+4x\right]_{-\frac{1}{2}}^{4}=$ 2

$=\frac{-128}{3}+56+16-\left(\frac{1}{12}+\frac{7}{8}-2\right)=\dots$ 2