3. VIZSGA AlaGIK 2024.06.11.

① $a_3 = 100$, $d = -10 \Rightarrow a_1 = a_3 - 2\cdot d = 120$ 1p

1p $a_n = a_1 + (n-1)d = 120 + (n-1)(-10)$

$a_n = -10n + 130$ $n \uparrow \Leftrightarrow -10n \downarrow \Leftrightarrow a_n$ szig.m.csökk. 1

$\lim\limits_{n\to\infty} \underbrace{-10n}_{\downarrow -\infty} + 130 = -\infty$ 1

$\Rightarrow a_n$ felülről korlátos, $\sup a_n = a_1 = 120$ 2

és a_n alulról NEM korl., $\inf a_n$ nem létezik. 2

② $f(x) = \begin{cases} \dfrac{x^2-6x-7}{2x^2+3x+1} & x \in \mathbb{R}\setminus\{-1; -\frac{1}{2}\} \\ 8 & x \in \{-1; -\frac{1}{2}\} \end{cases}$

Rac. törtfüggvény folyt., ahol értelmezve van. 1

$\dfrac{x^2-6x-7}{2x^2+3x+1} = \dfrac{\cancel{(x+1)}(x-7)}{2\cancel{(x+1)}(x+0{,}5)}$

$(x_{1,2} = \dfrac{-3 \pm \sqrt{9-8}}{4} \swarrow^{-1}_{-\frac{1}{2}}$

$\Rightarrow \mathbb{R}\setminus\{-1; -\frac{1}{2}\}$ halmazon f folytonos 1

$\lim\limits_{x\to -1} f(x) = \lim\limits_{x\to -1} \dfrac{x-7}{2x+1} = \dfrac{-8}{-1} = 8 = f(-1)$ 2p

tehát f a -1-ben is folytonos. 1p

$\lim\limits_{x\to -\frac{1}{2}-} f(x) = \lim\limits_{x\to -\frac{1}{2}-} \dfrac{x-7}{2x+1} = +\infty$ és $\lim\limits_{x\to -\frac{1}{2}+} \dfrac{x-7}{2x+1} = -\infty$ 2p

$" \dfrac{-7{,}5}{0\ominus} "$ $" \dfrac{-7{,}5}{0\oplus} "$

így f nem folyt. $x = \frac{-1}{2}$-ben, hanem pólusa van. 1

③ $x^2 + 3y^2 = 4$ $P(1; -1)$

$1^2 + 3\cdot(-1)^2 = 4$ ✓ P rajta van a görbén 2p

$()'_x$

$2x + 3\cdot 2\cdot y\cdot y' = 0$ 2p

$y' = \dfrac{-2x}{6y}$ ha $y \neq 0 \Rightarrow y'(1) = \dfrac{-2\cdot 1}{6\cdot(-1)} = \dfrac{1}{3}$

Érintő: $y = y'(x_0)\cdot(x - x_0) + y_0$ 2p

$y = \frac{1}{3}\cdot(x-1) - 1$ $\left(y = \frac{1}{3}x - \frac{4}{3}\right)$ 2p

4) $g(x) = \log_2(5x+1)$ $\qquad D_g:\ 5x+1>0$

$\boxed{x > -\frac{1}{5}}$ 2p

$g'(x) = \frac{1}{5x+1} \cdot \frac{5}{\ln 2}$ 2p

$g''(x) = -1 \cdot \frac{5}{(5x+1)^2} \cdot \frac{5}{\ln 2} = \frac{-25}{(5x+1)^2 \cdot \ln 2} < 0$ D_g-n 2p

$\Rightarrow g(x)$ D_g-n mindenütt konkáv 2p

5) a) $\int \sqrt{6x-1}\,dx = \int (6x-1)^{\frac{1}{2}}\,dx = \frac{(6x-1)^{\frac{3}{2}}}{\frac{3}{2}\cdot 6} + c =$ 1p

(lineáris helyettesítés tétele)

$= \frac{(6x-1)^{3/2}}{9} + c$ $\quad \boxed{c \in \mathbb{R}}$

2p 1p

b) $\int \frac{3x-2}{x^2+4x+3}\,dx = \int \frac{-5}{2}\cdot\frac{1}{x+1} + \frac{11}{2}\cdot\frac{1}{x+3}\,dx = \frac{-5}{2}\cdot\ln|x+1| + \frac{11}{2}\cdot\ln|x+3| + c$ 1p

$(c \in \mathbb{R})$ 1p

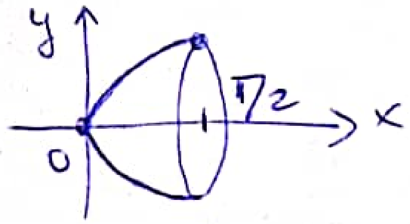
$\frac{3x-2}{(x+1)(x+3)} = \frac{a}{x+1} + \frac{b}{x+3} = \frac{a(x+3)+b(x+1)}{(x+1)(x+3)} = \frac{(a+b)x+(3a+b)}{x^2+4x+3}$

$\begin{cases} a+b = 3 \\ 3a+b = -2 \end{cases}$

$2a = -5$

$a = \frac{-5}{2}$ $\quad b = \frac{6}{2} - \left(-\frac{5}{2}\right) = \frac{11}{2}$ 2p

6) $f(x) = \sin(x)$ $\quad x \in \left[0; \frac{\pi}{2}\right]$



$V_{\text{forgást.}} = \pi \cdot \int_{x=0}^{\pi/2} \sin^2(x)\,dx =$ 2p

$= \pi \cdot \int_{x=0}^{\pi/2} \frac{1-\cos 2x}{2}\,dx = \frac{\pi}{2} \cdot \left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2} =$ 2p 2p

$= \frac{\pi}{2} \cdot \left(\frac{\pi}{2} - \underbrace{\frac{\sin \pi}{2}}_{=0} - \left(0 - \underbrace{\frac{\sin 0}{2}}_{=0}\right)\right) = \frac{\pi^2}{4}$

2p