4. VIZSGA Ala GTK 2024.06.18.

1) $\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$

a) $n=1$ $1\cdot 2 = \frac{1\cdot 2\cdot 3}{3}$ ✓

$n=2$ $1\cdot 2 + 2\cdot 3 = \frac{2\cdot 3\cdot 4}{3}$ ✓ 2p

b) Tfh. $n=m$-ig minden term. számra igaz.

$\sum_{k=1}^{m} k\cdot(k+1) = \frac{m(m+1)(m+2)}{3}$ 2p

c) ell. $(n=m+1)$-re

$B.O. = \sum_{k=1}^{m+1} k(k+1) = \sum_{k=1}^{m} k(k+1) + (m+1)(m+2) \overset{\text{ind. felt.}}{=}$

$= \frac{m(m+1)(m+2)}{3} + \frac{3(m+1)(m+2)}{3} = \frac{(m+1)(m+2)(m+3)}{3}$ 2p

$J.O.:$ $\frac{(m+1)(m+2)(m+3)}{3}$ 1p

d) $B.O. \equiv J.O. \iff$ az eredeti állítás igaz $\forall n \in \mathbb{N}^+$ esetén 1p

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2) a) $\lim_{n\to\infty} \frac{2\sqrt{n}+9}{n+1} = \lim_{n\to\infty} \frac{2+\frac{9}{\sqrt{n}}}{\sqrt{n}+\frac{1}{\sqrt{n}}} = 0$ 2p 1p

típus: $„\frac{2}{\infty}"$ 1p

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b) $\lim_{n\to\infty} \sqrt[n]{1-4^n+5^n} = 5$ mert:

$\underbrace{\sqrt[n]{\frac{1}{2}\cdot 5^n}}_{\frac{5}{\sqrt[n]{2}}} \le \sqrt[n]{1-4^n+5^n} \le \sqrt[n]{[redacted]\cdot 5^n} = 5$ 1p 1p

$\frac{5}{\sqrt[n]{2}} \xrightarrow{n\to\infty} \frac{5}{1} = 5$

Rendőrelv

$\sqrt[n]{1-4^n+5^n} \xrightarrow{n\to\infty} 5$ 1p

$\sqrt[n]{[redacted]\cdot 5^n} \xrightarrow{n\to\infty} 5$ 1p

3) $f(x) = x^x$ $\quad D_f = \mathbb{R}^+$ (2p) $\quad x_0 = 3$

$$f'(x) = \left(e^{x \cdot \ln x}\right)' = e^{x \cdot \ln x} \cdot \left(1 \cdot \ln(x) + x \cdot \frac{1}{x}\right) = x^x \left(\ln(x) + 1\right)$$ (2p)

$f'(3) = 27 \cdot (\ln(3) + 1)$ (1p) és $f(3) = 27$ (1p)

Érintő: $y = 27 \cdot (\ln 3 + 1) \cdot (x - 3) + 27$ (2p)

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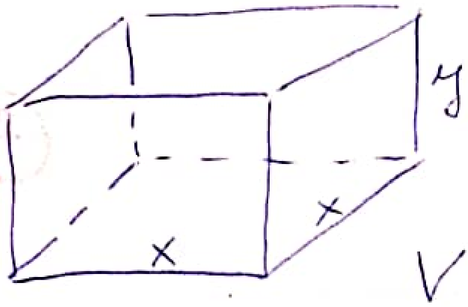
4)



$x^2 + 4xy = 75 \ (dm^2)$ (2p)

$$y = \frac{75 - x^2}{4x}$$

$$V(x) = x^2 \cdot \frac{75 - x^2}{4x} = \frac{75x - x^3}{4}$$ (1p)

$$V'(x) = \frac{75 - 3 \cdot x^2}{4}$$ (1p) $= 0 \Leftrightarrow x^2 = 25$ (1p) $x = 5 \ (dm)$

	$]0;5[$	$x=5$	$]5;\sqrt{75}[$
V'	+	0	−
V	↗	Lok MAX	↘

(2p)

$x_{max} = 5 \ (dm)$, $y_{max} = \frac{50}{20} = 2{,}5 \ (dm)$ (1p)

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5) a) $$\int \underbrace{(x-1)}_{f} \cdot \underbrace{\sin(2x)}_{g'} dx = \frac{-(x-1)\cos(2x)}{2} + \frac{1}{2}\int \cos(2x)\, dx = (*)$$ (1p)

$f'(x) = 1$, $g(x) = \frac{-\cos(2x)}{2}$ (1p)

$$(*) = \frac{-(x-1) \cdot \cos(2x)}{2} + \frac{\sin(2x)}{4}$$ (1p) $+ c$ $\quad c \in \mathbb{R}$ (1p)

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b) $$\int \frac{3}{4 + 9x^2} dx = \frac{3}{4} \cdot \int \frac{1}{1 + \frac{9}{4}x^2} dx$$ (1p) $$= \frac{3}{4} \cdot \int \frac{1}{1 + \left(\frac{3}{2}x\right)^2} dx =$$ (1p)

$$= \frac{3}{4} \cdot \frac{\text{arctg}\left(\frac{3}{2}x\right)}{\frac{3}{2}} + c = \frac{\text{arctg}\left(\frac{3}{2}x\right)}{2} + c$$ (1p)

$c \in \mathbb{R}$ (1p)

⑥ $f(x) = 4x + 1 \qquad x \in [0; 3]$

$$A_{palást} = 2\pi \cdot \int_{x=0}^{3} (4x+1) \cdot \sqrt{1+(4)^2}\, dx = \quad (2p)$$

$$= 2 \cdot \sqrt{17} \cdot \pi \cdot \left[2x^2 + x\right]_0^3 = 2 \cdot \sqrt{17} \cdot \pi \cdot (21 - 0) = \quad (1p)$$

$$= 42 \cdot \sqrt{17} \cdot \pi \quad (1p) \quad //$$

$$V_{forg.test} = \pi \cdot \int_{x=0}^{3} (4x+1)^2\, dx = \pi \cdot \int_{x=0}^{3} 16x^2 + 8x + 1\, dx = \quad (2p)$$

$$= \pi \cdot \left[\frac{16}{3}x^3 + 4x^2 + x\right]_0^3 = \pi \cdot (\underbrace{16 \cdot 9 + 4 \cdot 9}_{20 \cdot 9} + 3 - 0) = \quad (1p)$$

$$= 183 \cdot \pi \quad (1p) \quad //$$

CamScannerrel szkennelve