Konzultáció - A1a GTK, 2024. június 10.

① $a_n = \frac{n+4}{3-n} - 3$

a) $a_{n+1} \lesseqgtr a_n$

b) $a_{n+1} - a_n \lesseqgtr 0$

c) ha a_n POZITÍV

$$\frac{a_{n+1}}{a_n} \lesseqgtr 1$$

$$a_n = \frac{1 + \frac{4}{n}}{\frac{3}{n} - 1} - 3 \xrightarrow[n \to \infty]{} \frac{1}{-1} - 3 = -4$$

$\frac{4}{n} \to 0$, $\frac{3}{n} \to 0$

a_n konvergens

$\Downarrow$

a_n korlátos is

$$a_{n+1} \overset{\leq}{\underset{>}{=}} a_n \iff \frac{n+5}{2-n} - \cancel{3} \overset{\leq}{\underset{>}{=}} \frac{n+4}{3-n} - \cancel{3}$$

$$\frac{n+5}{2-n} - \frac{n+4}{3-n} \overset{\leq}{\underset{>}{=}} 0$$

$$\frac{(n+5)(3-n) - (n+4)(2-n)}{(2-n)\cdot(3-n)} \overset{\leq}{\underset{>}{=}} 0$$

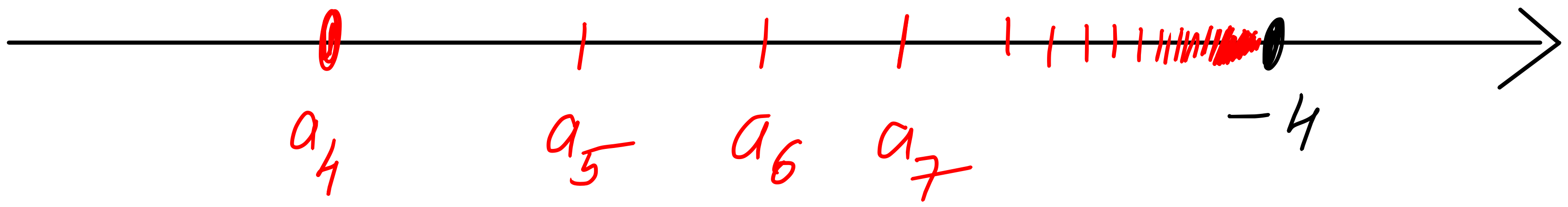
$n \geq 4$

$$\frac{\cancel{-n^2} \cancel{-2n} + 15 - (\cancel{-n^2} \cancel{-2n} + 8)}{(2-n)(3-n)} \overset{\leq}{\underset{>}{=}} 0 \quad +$$

$$\frac{+\ 7}{+\ (2-n)(3-n)} \overset{\leq}{\underset{>}{=}} 0$$

$\forall n \, (\geq 4, \in \mathbb{N}) \quad a_{n+1} - a_n > 0 \iff a_{n+1} > a_n$

$\overset{\text{Def.}}{\iff}$ szig. mon. növő sorozat



$$\inf(a_n) = a_4 = \min a_n = \frac{4+4}{3-4} - 3 = -8-3 = -11$$

$\max a_n$ nem létezik, mert szig. mon. növő a sorozat

$$\sup(a_n) = \lim_{n\to\infty} a_n = -4$$

(2) $x^4 - x^2 - 12 = P(x)$

$\pm 1 \quad \pm 2 \quad \pm 3 \quad \pm 4 \quad \pm 6 \quad \pm 12$

$x_1 = 2 \quad x_2 = -2$ gyökök

$(x-2)(x+2) = x^2 + 0x - 4$

$x^4 + 0x^3 - x^2 + 0x - 12 : (x^2 + 0x - 4) = \ldots$

$x^{2k} - x^k - 12$

$(x^k)^2 - x^k - 12$

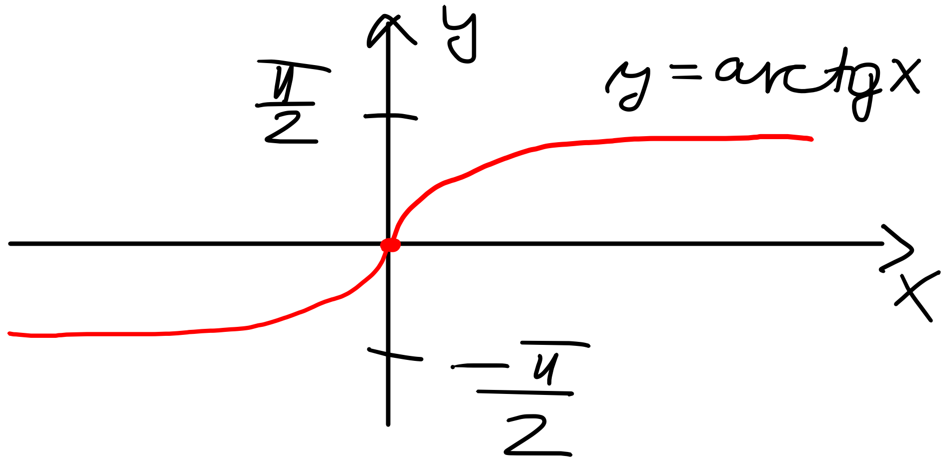
$x^k = y$

$y^2 - y - 12 = 0$

$y = \dfrac{1 \pm \overbrace{\sqrt{1+48}}^{7}}{2}$ $\quad y_1 = -3, \quad y_2 = 4$

$x^k = -3 \ldots$ ill. $x^k = 4 \ldots$

③ $$f(x)=\begin{cases}\operatorname{arctg}\frac{1}{x} & x\in\mathbb{R}\setminus\{0;1\}\\[4pt] \frac{\pi}{2} & x\in\{0;1\}\end{cases}$$



$\operatorname{arctg}\frac{1}{x}$ $\forall x\in\mathbb{R}\setminus\{0\}$ folytonos amúgy

Ellenőrizni a 0 és 1 pontban kell.

$$\left.\begin{array}{l}\lim\limits_{x\to 0-}\operatorname{arctg}\underbrace{\frac{1}{x}}_{\to-\infty}=-\frac{\pi}{2}\\[16pt] \lim\limits_{x\to 0+}\operatorname{arctg}\underbrace{\frac{1}{x}}_{\to+\infty}=+\frac{\pi}{2}\end{array}\right\}$$ f-nek $x=0$-ban ugrása van

$$\lim_{x \to 1} \operatorname{arctg} \frac{1}{x} = \operatorname{arctg} 1 = \frac{\pi}{4} \neq \frac{\pi}{2}$$

$$\lim_{x \to 1} \operatorname{arctg} \frac{1}{x} = \frac{\pi}{4} \neq f(1) = \frac{\pi}{2}$$

f-nek $x=1$-ben megszüntethető szakadása van

④ $\sqrt{x+1} < \frac{1}{2}x - 1 \quad /()^2$ Alaph.: $x+1 \geq 0$ és $\frac{x}{2} - 1 > 0$

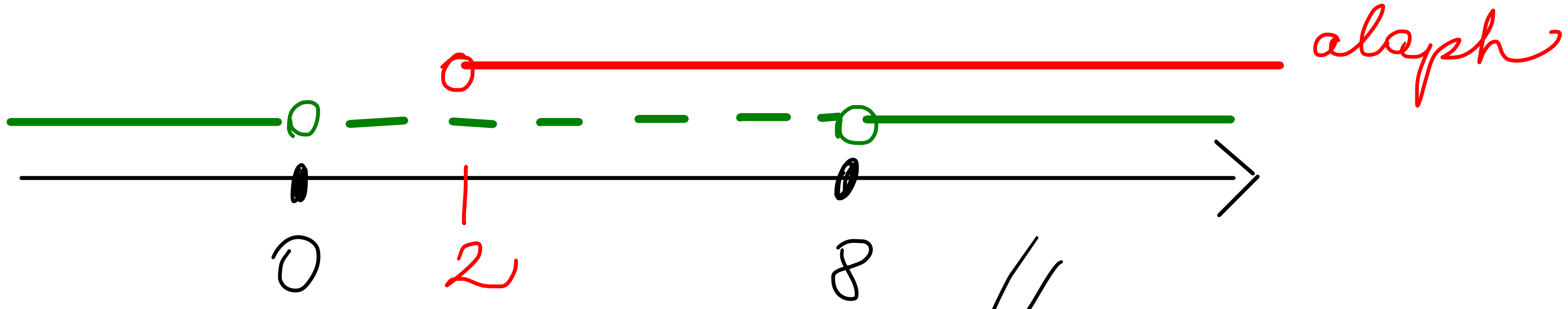
$x \geq -1$ és $x > 2$

$\cancel{x+1} < \frac{x^2}{4} - x + \cancel{1}$

$\Updownarrow$

$0 < \frac{x^2}{4} - 2x$

$0 < \frac{1}{4}x(x-8)$



M.O. $= \left]8; \infty\right[$

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$$\int \underbrace{x^2}_{f} \cdot \underbrace{\sin(3x)}_{g'}\, dx = -\frac{1}{3} x^2 \cdot \cos(3x) + \frac{2}{3}\int \underbrace{x}_{f} \cdot \underbrace{\cos(3x)}_{g'}\, dx =$$

$$f'(x) = 2x \qquad g(x) = \frac{-\cos(3x)}{3} \qquad\qquad f'(x) = 1 \qquad g(x) = \frac{\sin(3x)}{3}$$

$$= -\frac{1}{3} x^2 \cdot \cos(3x) + \frac{2}{3} \cdot \left(\frac{x \cdot \sin 3x}{3} - \frac{1}{3}\int \sin(3x)\, dx \right) =$$

$$= -\frac{1}{3} \cdot x^2 \cdot \cos(3x) + \frac{2}{9} \cdot x \cdot \sin(3x) + \frac{2}{9} \cdot \frac{+\cos(3x)}{3} + C$$

$$C \in \left]-\infty ; \infty\right[$$

⑥ $\int \underbrace{(x+1)}_{g'} \cdot \underbrace{\ln(x)}_{f} \cdot dx = \left(\frac{x^2}{2}+x\right)\cdot \ln(x) - \int \frac{x}{2} + 1 \, dx =$

$\underbrace{f'(x) = \frac{1}{x} \quad , \quad g(x) = \frac{x^2}{2} + x}$

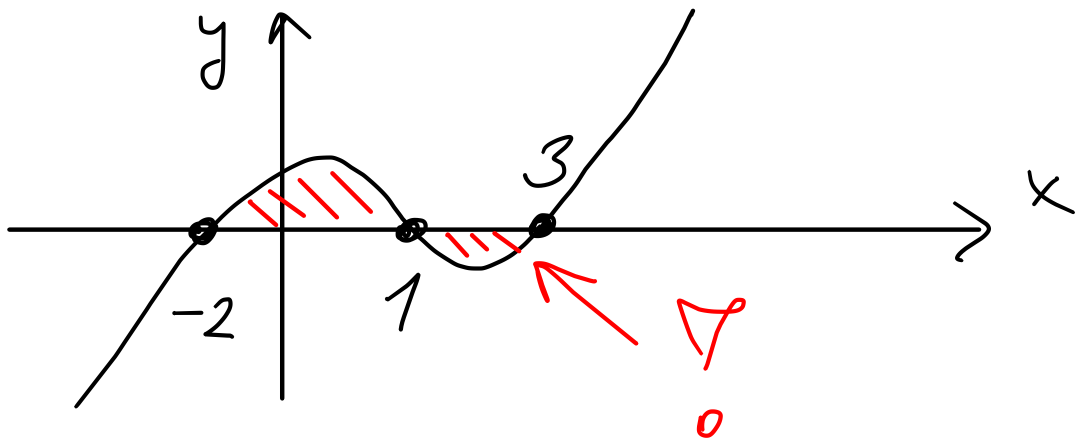
$f'(x) \cdot g(x) = \frac{1}{x}\left(\frac{x^2}{2} + x\right)$

$= \left(\frac{x^2}{2}+x\right)\cdot \ln(x) - \left(\frac{x^2}{4} + x\right) + C \qquad C \in \,]-\infty, \infty[$

(7) $f(x) = x^3 - 2x^2 - 5x + 6$

$\pm 1, \pm 2, \pm 3, \pm 6$

$x_1 = 1$ gyök



$$(x^3 - 2x^2 - 5x + 6) : (x - 1) = \underbrace{x^2 - x - 6}$$

$$\begin{array}{l} \ominus\ \underline{x^3 - x^2} \\ \quad -x^2 - 5x \\ \ominus\ \underline{-x^2 + x} \\ \qquad -6x + 6 \\ \quad \ominus\ \underline{-6x + 6} \\ \qquad\qquad 0 \end{array}$$

$$x_{2,3} = \frac{1 \pm \overset{5}{\sqrt{1+24}}}{2} \quad \begin{array}{l} x_2 = -2 \\ x_3 = 3 \end{array}$$

$$T = \int_{x=-2}^{1} x^3 - 2x^2 - 5x + 6\,dx + \left| \int_{x=1}^{3} x^3 - 2x^2 - 5x + 6\,dx \right| = \text{stb...}$$