Konzultáció - A1a GTK 2024. június 17.

① $\int \sqrt{1+x^2}\, dx = *$

$$\begin{cases} x = sh(t) \quad t \in \mathbb{R} \quad arsh\, x = t \\ x' = x'(t) = \frac{dx}{dt} = ch(t) \iff dx = ch(t) \cdot dt \end{cases}$$

$$\begin{cases} ch^2(t) - sh^2(t) = 1 \\ ch^2(t) = \frac{ch(2t)+1}{2} \end{cases}$$

$$* = \int \sqrt{1+sh^2(t)} \cdot ch(t) \cdot dt = \int ch(t) \cdot ch(t) \cdot dt = \int ch^2(t) \cdot dt =$$

$$= \frac{1}{2} \cdot \int ch(2t) + 1\, dt = \frac{1}{2} \cdot \left(\frac{sh(2t)}{2} + t \right) + c \quad =$$

$$c \in]-\infty; \infty[$$

$$= \frac{1}{2} \cdot \left(\frac{sh(2 \cdot arsh\, x)}{2} + arsh\, x \right) + c$$

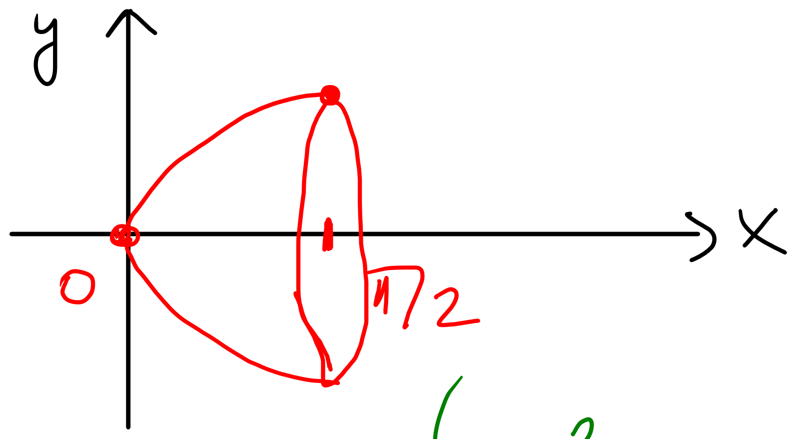
② $\int x^2\cdot e^{-x^3}\,dx = \frac{-1}{3}\cdot\int -3x^2\cdot e^{-x^3}\cdot dx = \frac{-1}{3}\cdot e^{-x^3} + c \qquad c\in\mathbb{R}$

$g'(x)\cdot f(g(x))$

$(-x^3)' = -3\cdot x^2$

$(F(g(x)))' = (e^{-x^3})' = e^{-x^3}\cdot(-3x^2)$

③ $f(x) = \sin(x) \qquad x\in[0;\pi/2]$



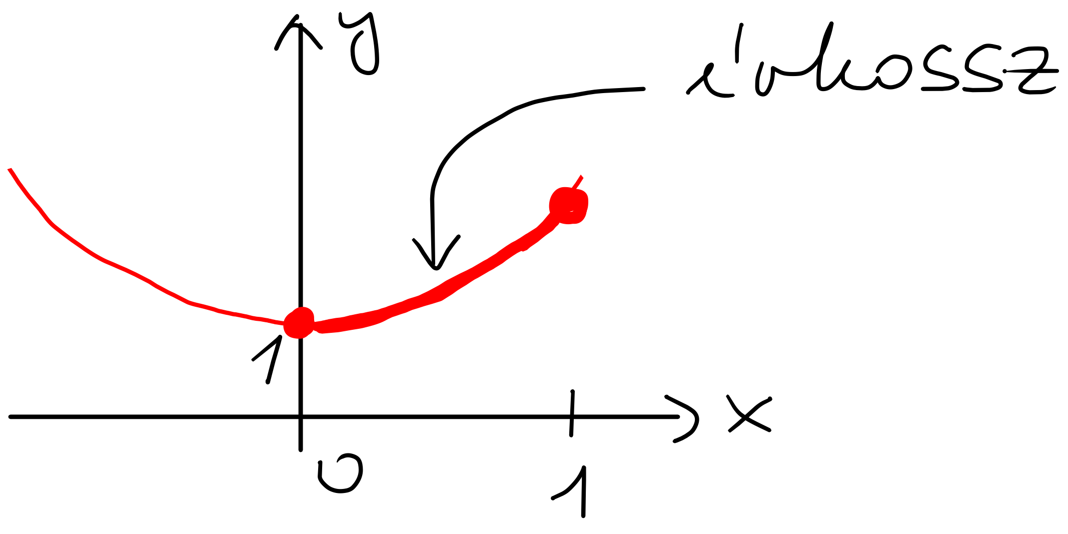
$$V_{f.t.} = \pi\cdot\int_{x=a}^{b} f^2(x)\,dx =$$

$$\left(\sin^2(x) = \frac{1-\cos 2x}{2}\right)$$

$$= \pi\cdot\int_{x=0}^{\pi/2}\sin^2(x)\,dx = \pi\cdot\int_{x=0}^{\pi/2}\frac{1-\cos 2x}{2}\,dx = \frac{\pi}{2}\cdot\left[x - \frac{\sin(2x)}{2}\right]_0^{\pi/2} =$$

$$= \frac{\pi}{2}\cdot\left(\frac{\pi}{2} - \frac{\underset{\overset{\|}{0}}{\sin\pi}}{2} - \left(0 - \frac{\underset{\overset{\|}{0}}{\sin 0}}{2}\right)\right) = \frac{\pi}{2}\cdot\frac{\pi}{2} = \frac{\pi^2}{4}$$

4. $f(x)=ch(x) \qquad x\in[0;1]$



$$I=\int_{x=0}^{1}\sqrt{1+\underbrace{(sh\,t)^2}_{(f'(x))^2}}\,dx =$$

$$=\int_{x=0}^{1}\sqrt{ch^2(x)}\,dx =$$

$$ch^2(x)-sh^2(x)=1$$

$$\underline{\underline{ch^2(x)=1+sh^2(x)}}$$

$$=\int_{x=0}^{1}|ch(x)|\,dx=\int_{x=0}^{1}ch(x)\,dx=\Big[sh(x)\Big]_0^1=sh1-\underbrace{sh0}_{=0}=sh1=$$

$$=\frac{e^1-e^{-1}}{2}$$

⑤ $\overbrace{\int \underbrace{4^x}_{f}\cdot \underbrace{\sin(x)}_{g'}\cdot dx}^{I} = -4^x\cdot\cos(x) + \int 4^x\cdot\cos(x)\cdot \ln 4\, dx =$

$f' = 4^x\cdot\ln 4 \qquad g = -\cos x$

$= -4^x\cdot\cos(x) + \ln 4\cdot\int \underbrace{4^x}_{f}\cdot\underbrace{\cos(x)}_{g'}\,dx = -4^x\cdot\cos x + \ln 4\cdot\left(4^x\cdot\sin x - \ln 4\cdot\underbrace{\int 4^x\cdot\sin x\,dx}_{I}\right)$

$f' = 4^x\cdot\ln 4 \qquad g = \sin x$

$\Leftrightarrow I = -4^x\cdot\cos x + \ln 4\cdot 4^x\cdot\sin x - (\ln 4)^2\cdot I$

$(1+(\ln 4)^2)\cdot I = -4^x\cdot\cos x + \ln 4\cdot 4^x\cdot\sin x + C \qquad C\in\,]-\infty,\infty[$

$\Leftrightarrow \int 4^x\cdot\sin x\,dx = \frac{1}{1+(\ln 4)^2}\cdot 4^x\cdot\left(-\cos x + \ln 4\cdot\sin x\right) + k \qquad k\in\,]-\infty,\infty[$

⑥ $g(x) = \log_2(5x+1)$

$D_g: \ 5x+1 > 0$

$x > \frac{-1}{5}$

$$g'(x) = \frac{5}{(5x+1)\cdot \ln 2} = \frac{5}{\ln 2}\cdot(5x+1)^{-1}$$

$$g''(x) = \frac{5}{\ln 2}\cdot(-1)\cdot(5x+1)^{-2}\cdot 5 = \frac{\overbrace{-25}^{\ominus}}{\underbrace{\ln 2}_{\oplus}}\cdot\underbrace{\frac{1}{(5x+1)^2}}_{\oplus} < 0$$

a teljes D_g-on

$\Downarrow$

	$]-\frac{1}{5}; \infty[$
g''	$-$
g	konkáv ($\frown$)

$g(x)$ konkáv a D_g-on ($\frown$)

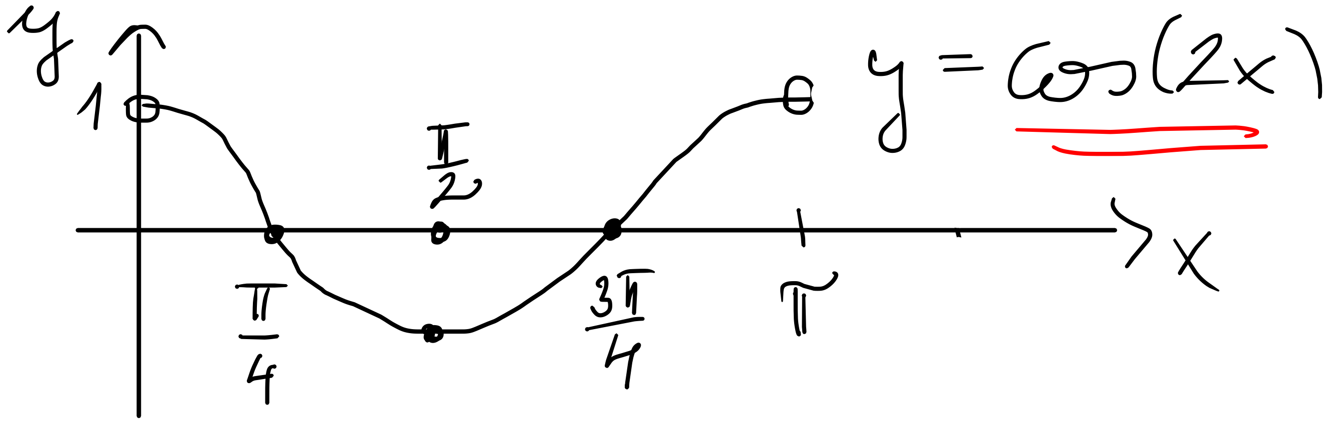
⑦ $g(x) = \cos(2x) \qquad x \in]0;\pi[\qquad D_g = \mathbb{R}$

$g'(x) = -\sin(2x)\cdot 2 = -2\cdot\sin(2x)$

$g''(x) = -4\cos(2x)$

	$]0;\frac{\pi}{4}[$	$\frac{\pi}{4}$	$]\frac{\pi}{4};\frac{3\pi}{4}[$	$\frac{3\pi}{4}$	$]\frac{3\pi}{4};\pi[$
g''	$-$	0	$+$	0	$-$
g	konkáv $\frown$	IP	konvex $\smile$	IP	$\frown$

$\cos(2x)$



(8)

$$k(x) = 3x - \sqrt[3]{x} = \qquad D_k = \mathbb{R}$$

$$= 3x - x^{1/3}$$

$$k'(x) = 3 - \frac{1}{3} \cdot x^{-\frac{2}{3}} = 3 - \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}} \qquad x \neq 0$$

$$= \frac{9 \cdot \sqrt[3]{x^2} - 1}{3 \cdot \sqrt[3]{x^2}} \;(+) = 0 \iff 9 \cdot \sqrt[3]{x^2} - 1 = 0$$

$$\sqrt[3]{x^2} = \frac{1}{9}$$

$$\sqrt[3]{x} = \pm \frac{1}{3}$$

$$x = \pm \frac{1}{27}$$

	$]-\infty; -\frac{1}{27}[$	$-\frac{1}{27}$	$]-\frac{1}{27}; 0[$	0	$]0; \frac{1}{27}[$	$\frac{1}{27}$	$]\frac{1}{27}; \infty[$
k'	+	0	−	////	−	0	+
k	sz. mon. nő	LOK. MAX.	sz. mon. csökk	$\searrow$	sz. m. csökk.	LOK. MIN	szig. mon. nő

$$f\left(\frac{-1}{27}\right) = \frac{-1}{9} + \frac{1}{3} = \frac{2}{9}$$

$$f\left(\frac{1}{27}\right) = \frac{1}{9} - \frac{1}{3} = \frac{-2}{9}$$

9 $$\int \frac{2x+1}{x^2-x+3}\,dx = \int \frac{2x-1+2}{x^2-x+3}\,dx = *$$

$$x_{1,2} = \frac{1 \pm \sqrt{1-12}}{2}$$

$D = 1-12 < 0$ nincs valós gyök

$$x^2-x+3 = \left(x-\frac{1}{2}\right)^2 + \underbrace{2{,}75}_{2+\frac{3}{4}=\frac{11}{4}}$$

$$(x^2-x+3)' = 2x-1$$

$$* = \int \underbrace{\frac{2x-1}{x^2-x+3}}_{\frac{f'(x)}{f(x)}} + \frac{2}{x^2-x+3}\,dx = \ln|x^2-x+3| + \int \frac{2}{\frac{11}{4}+\left(x-\frac{1}{2}\right)^2}\,dx =$$

$$= \ln|x^2-x+3| + \frac{8}{11}\cdot\int \frac{1}{1+\underbrace{\frac{4}{11}\left(x-\frac{1}{2}\right)^2}_{\left(\frac{2}{\sqrt{11}}\cdot x-\frac{1}{\sqrt{11}}\right)^2}}\,dx = \ln|x^2-x+3| + \frac{8}{11}\cdot\frac{\operatorname{arctg}\left(\frac{2}{\sqrt{11}}x-\frac{1}{\sqrt{11}}\right)}{\frac{2}{\sqrt{11}}} + C$$

$C \in \mathbb{R}$