KONZULTÁCIÓ A1a GTK 2024. június 24.

1. $\begin{cases} x+y=10 \\ x^3+y^3 \text{ minimális} \end{cases}$ $\quad x,y>0$

$y=10-x \xRightarrow{\text{beh.}} x^3+(10-x)^3 = f(x) \quad \text{min. ?}$

$$3x^2+3\cdot(10-x)^2\cdot(-1) = f'(x)$$

$$\cancel{3x^2}-3(\cancel{x^2}-20x+100) = f'(x)$$

$$60x-300 = \boxed{f'(x)} = 0 \iff 60x=300$$

$$x=5$$

	$]0;5[$	$x=5$	$]5;10[$
f'	$-$	0	$+$
f	szig. mon. cs. ↘	LOK MIN	szig. mon. növő ↗

f min. $\iff x=5$, $y=5$

② $f(x) = x^2 \cdot e^x$

1) $D_f = \mathbb{R}$

2) zh: $f(x) = 0 \iff x^2 = 0$, $\boxed{x = 0}$

3) nem period., paritás: $(-x)^2 \cdot e^{-x} = \dfrac{x^2}{e^x} = f(-x)$ nem ps, nem ptlan

4) $\lim\limits_{x\to-\infty} x^2 \cdot e^x = \lim\limits_{x\to-\infty} \dfrac{x^2}{e^{-x}} \overset{\text{L'Hospital}}{=} \lim\limits_{x\to-\infty} \dfrac{2x}{-e^{-x}} \overset{\text{L'Hosp}}{=} \lim\limits_{x\to-\infty} \dfrac{2}{e^{-x}} = 0$

"$\infty \cdot 0$" "$\frac{-\infty}{-\infty}$" "$\frac{2}{\infty}$"

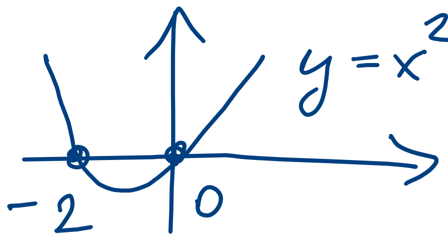
vízszintes asz. $-\infty$ -ben

$\lim\limits_{x\to\infty} x^2 \cdot e^x = \infty$

"$\infty \cdot \infty$"

Ferde asz. $+\infty$ -ben nincs, e^x és x^2 is meredekebb, mint egy egyenes

5) $f'(x) = 2x \cdot e^x + x^2 \cdot e^x = x(2+x) \cdot e^x = 0 \iff x = 0$ VAGY $x = -2$



	$x < -2$	-2	$]-2;0[$	0	$x > 0$
f'	+	0	—	0	+
f	sz.m.nő ↗	LOK MAX	sz.m. csökk ↘	LOK MIN	↗

$f(-2) = 4 \cdot e^{-2} = \frac{4}{e^2}$

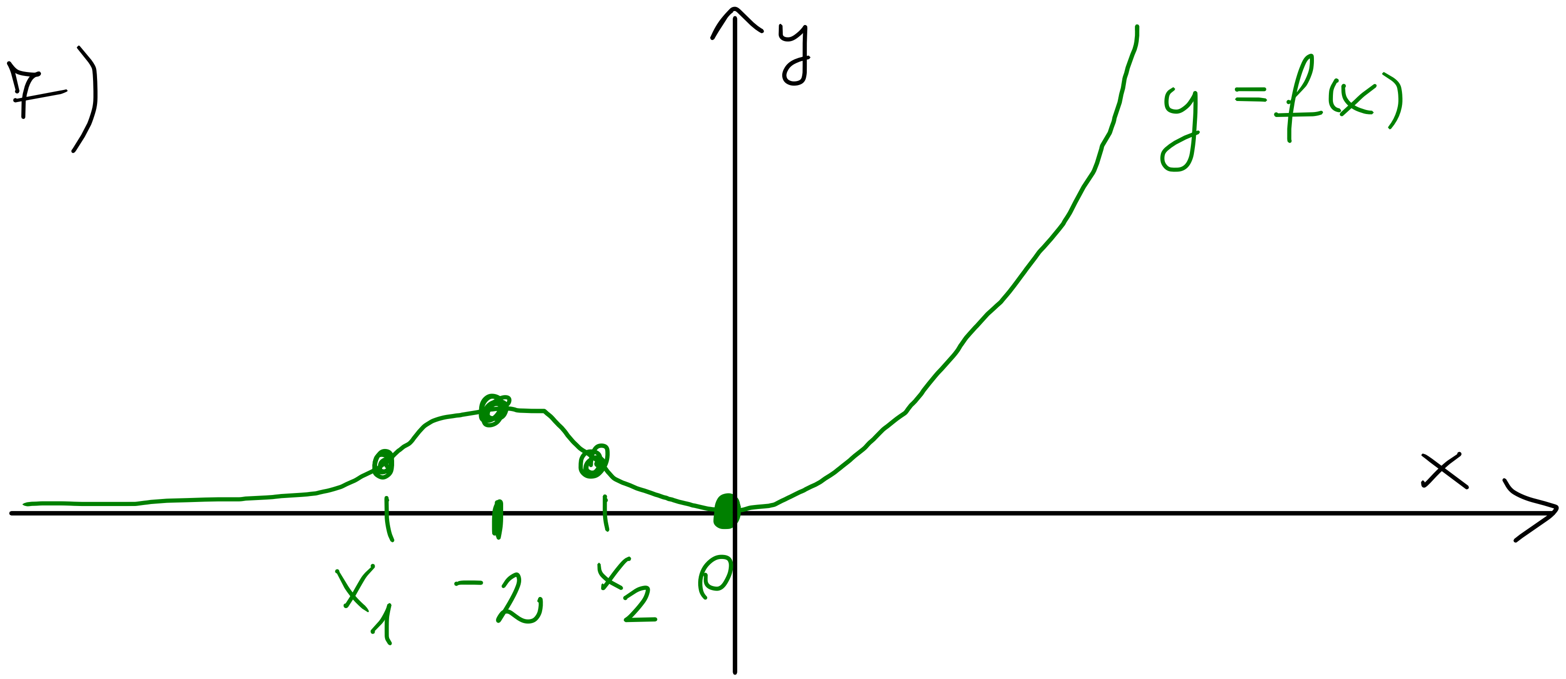
$f(0) = 0$

6) $f''(x) = \left[(x^2+2x)\cdot e^x\right]' = (2x+2)\cdot e^x + (x^2+2x)\cdot e^x = \underbrace{e^x}_{+}\cdot(x^2+4x+2)$

$$x_{1,2} = \frac{-4 \pm \sqrt{16-8}}{2} = \frac{-4 \pm 2\cdot\sqrt{2}}{2} \quad \begin{cases} -2-\sqrt{2} = x_1 \\ -2+\sqrt{2} = x_2 \end{cases}$$

	$x < x_1$	$-2-\sqrt{2}$	$]x_1; x_2[$	$-2+\sqrt{2}$	$x_2 < x$
f''	+	0	—	0	+
f	konvex	IP	konkáv	IP	

7)



8) $R_f = [0; \infty[$

③ $a_n = \dfrac{\sin(n^2)}{7n-2}$ Tip.: "$\dfrac{\text{korl.}}{+\infty}$"

$$\underbrace{\frac{-1}{7n-2}}_{\substack{\downarrow\, n\to\infty \\ 0}} \leq \frac{\sin(n^2)}{7n-2} \leq \underbrace{\frac{1}{7n-2}}_{\substack{\downarrow\, n\to\infty \\ 0}}$$

"$\dfrac{-1}{+\infty}$" "$\dfrac{1}{+\infty}$"

Rendőr-elv

$n\to\infty$

0

④ $\int f \cdot g' = f \cdot g - \int f' \cdot g$

$$\int (\underbrace{x^2+x-5}_{f}) \cdot \underbrace{\cos(x)}_{g'} dx = (x^2+x-5) \cdot \sin(x) - \int (\underbrace{2x+1}_{f}) \cdot \underbrace{\sin(x)}_{g'} \cdot dx =$$

$f'(x) = 2x+1 \quad g = \sin(x) \qquad f'(x) = 2 \quad g(x) = -\cos(x)$

$$= (x^2+x-5) \cdot \sin(x) - \left(-(2x+1)\cos(x) + 2 \cdot \underbrace{\int \cos x \cdot dx} \right) =$$

$$= (x^2+x-5) \cdot \sin(x) + (2x+1) \cdot \cos(x) - 2 \cdot \sin(x) + C \qquad C \in]-\infty ; \infty[$$

⑤ $$\int \underbrace{(x^2+x-5)}_{g'} \cdot \underbrace{\ln(x)}_{f} \cdot dx = \left(\frac{x^3}{3}+\frac{x^2}{2}-5x\right)\cdot \ln(x) - \int \frac{x^2}{3}+\frac{x}{2}-5\,dx$$

$$= \ldots$$

$g(x) = \frac{x^3}{3}+\frac{x^2}{2}-5x \qquad f'(x) = \frac{1}{x}$

⑥ $$\int x \cdot \sin\left(\frac{x^2}{2}\right) dx = -\cos\left(\frac{x^2}{2}\right) + c$$

$g' \cdot f(g)$

$c \in \left]-\infty; \infty\right[$

$g(x) = \frac{x^2}{2}$

$g'(x) = x$

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$y = y(x)$ $\qquad$ $P(\overset{x_0}{1}; \overset{y_0}{-1})$

$x^2 + 3y^2 = 4$ $\qquad$ $1^2 + 3\cdot(-1)^2 = 4$ ✓ P rajta van a görbén

$()'_x$

$$2x + 3\cdot 2\cdot y^1 \cdot y' = 0$$

$$6yy' = -2x$$

$$y' = \frac{-2x}{6y} = \frac{-1}{3}\cdot\frac{x}{y}$$

$$y'\big|_P = \frac{-1}{3}\cdot\frac{1}{-1} = \frac{1}{3} \qquad (y'(x_0))$$

Érintő egyenes egyenlete: $\qquad y = f'(x_0)\cdot(x - x_0) + f(x_0)$

$$y = y'(x_0)\cdot(x - x_0) + y(x_0)$$

$y(x_0) = y_0$

$$y = \frac{1}{3}\cdot(x-1) - 1$$

(8) $\int e^{\frac{-3x+5}{2}} \cdot dx = \int e^{-\frac{3}{2}x+\frac{5}{2}} dx = \dfrac{e^{-\frac{3}{2}x+\frac{5}{2}}}{-\frac{3}{2}} + C$

lin. helyett. tétele

$C \in \mathbb{R}$

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + C$$

$a \neq 0$

(9) $\int \dfrac{\sin(\operatorname{arctg} x)}{3x^2+3} dx = \dfrac{1}{3} \cdot \int \underbrace{\sin(\operatorname{arctg}(x))}_{f(g(x))} \cdot \underbrace{\dfrac{1}{1+x^2}}_{g'(x)} \cdot dx =$

$= \dfrac{1}{3} \cdot \left(-\cos(\operatorname{arctg} x)\right) + C \qquad C \in \mathbb{R}$