$$\int \frac{x+4}{x^2+5x+6}\,dx$$

1. Polinomosztás?
számláló foka $\geq$ nevező foka
NEM KELL most

2. nevező gyökei $\quad x^2+5x+6=(x+2)^1(x+3)^1$

$x_1=-2 \quad x_2=-3$

3. $$\frac{x+4}{x^2+5x+6}=\frac{A}{x+2}+\frac{B}{x+3}$$

$$(x-\alpha)^k \to \frac{A}{x-\alpha}+\frac{B}{(x-\alpha)^2}+\dots+\frac{}{(x-\alpha)^k}$$

$$(x^2+\beta x+\gamma)^l \to \frac{Ax+B}{x^2+\beta x+\gamma}+\dots+\frac{}{(\ )^l}$$

bevezetjük a résztörteket

$\Downarrow$

közös nevező

$$\frac{x+4}{x^2+5x+6}=\frac{A(x+3)+B(x+2)}{(x+2)(x+3)}=\frac{Ax+3A+Bx+2B}{(x+2)(x+3)}$$

$$x+4=Ax+3A+Bx+2B=(A+B)x+3A+2B$$

$A+B=1 \quad B=1-A$

$3A+2B=4 \quad 3A+2-2A=4$

$A+2=4 \quad A=2 \quad B=-1$

$$\frac{x+4}{x^2+5x+6} = \frac{2}{x+2} + \frac{-1}{x+3}$$

4. Integrálás

$$\int \frac{2}{x+2} + \frac{-1}{x+3}\, dx = 2\int \frac{1}{x+2}\, dx - \int \frac{1}{x+3}\, dx =$$

$$= 2\ln|x+2| - \ln|x+3| + C$$

$$\int \frac{1}{x}\, dx = \ln|x| + c$$

$$\int \frac{1}{ax+b}\, dx = \frac{\ln|ax+b|}{a} + c$$

$$\int \frac{2}{x+2}\,dx = 2\ln|x+2| + C$$

$$\int \frac{2}{(x+2)^{3}}\,dx = 2\cdot\int (x+2)^{-3}\,dx = 2\cdot\frac{(x+2)^{-2}}{-2} + C = \frac{-1}{(x+2)^2} + C$$

-3

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int (x-\alpha)^n\,dx = \frac{(x-\alpha)^{n+1}}{n+1} + C$$

$$\int \frac{1}{x^2+4}\,dx \qquad x^2+4=0$$

$x^2=-4$ nincs gyök

$$\int \frac{1}{x^2+1}\,dx = \operatorname{arctg} x + C$$

$$\int \frac{1}{x^2+4}\,dx = \frac{1}{4}\cdot\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx = \frac{1}{4}\cdot\frac{\operatorname{arctg}\left(\frac{x}{2}\right)}{\frac{1}{2}} + C =$$

$$= \frac{1}{2}\cdot\operatorname{arctg}\left(\frac{x}{2}\right) + C$$

$$\int \frac{1}{x^2+4}\,dx \rightarrow \int \frac{x+1}{x^2+4}\,dx = \frac{1}{2}\int \frac{1}{x^2+4}\cdot 2x\,dx + \int \frac{1}{x^2+4}\,dx$$

$$f(g(x))\cdot g'(x) = \frac{1}{(g(x))}\cdot g'(x) \qquad \int f(x)\,dx =$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$g(x) = x^2+4$$

$$g'(x) = 2x$$

$$\frac{1}{2}\cdot \int \frac{1}{x^2+4}\cdot 2x\,dx = \frac{1}{2}\ln|x^2+4| + C$$

$$\int \frac{x+1}{x^2+4}\,dx = \frac{1}{2}\ln|x^2+4| + \frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right) + C$$

$$\lim_{x \to \infty} 2\ln(x-3) - (x-3)^2 = \text{"}\infty - \infty\text{"} \;\rightarrow\; = -\infty$$

snip indalla's

$$\ln(\cancel{x}n) \ll \cancel{x}n \ll \cancel{x}n^2 \ll 2^{\cancel{x}n}$$

$n \to \infty$

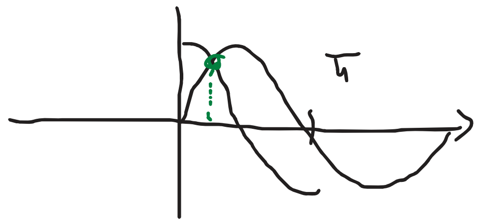
$$\lim_{\cancel{x}n \to \infty} \frac{\ln(\cancel{x}n)}{\cancel{x}n^2} = 0$$

$n \to \infty$

$$\lim_{\cancel{x}n \to \infty} \frac{\cancel{x}n^2}{\ln(\cancel{x}n)} = \infty$$

Mintavizsga 3.

$f(x) = \operatorname{arctg}(2x) \qquad x_0 = \frac{1}{2}$ érintő



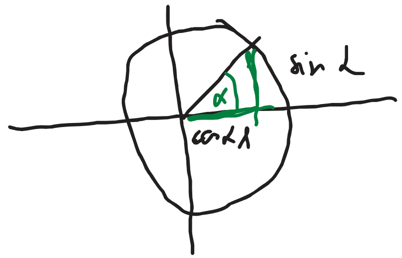
$$y = f'(x_0)(x - x_0) + f(x_0)$$

$x_0 = \frac{1}{2}$

$$f\left(\tfrac{1}{2}\right) = \operatorname{arctg}\left(2\cdot\tfrac{1}{2}\right) = \underbrace{\operatorname{arctg}(1)}_{\frac{\pi}{4}} \longrightarrow \operatorname{tg}\alpha = 1 = \frac{\sin\alpha}{\cos\alpha}$$

$$\alpha = \frac{\pi}{4} \ (45^\circ)$$

$$f'(x) = \left(\operatorname{arctg}(2x)\right)' = \frac{1}{(2x)^2+1}\cdot 2 = \frac{2}{4x^2+1}$$



$$f'\left(\tfrac{1}{2}\right) = \frac{2}{4\left(\frac{1}{2}\right)^2+1} = \frac{2}{1+1} = 1$$

$$y = 1\cdot\left(x - \frac{1}{2}\right) + \frac{\pi}{4} = x - \frac{1}{2} + \frac{\pi}{4}$$

Mintafeladat 2/a

$$\lim_{n\to\infty} \sqrt[n]{3^n - 2^n} \qquad \lim_{n\to\infty} \sqrt[n]{n} = 1$$

Rendőr-elv.

$$\underbrace{\sqrt[n]{3^n - \frac{3^n}{2}}} \leq \sqrt[n]{3^n - 2^n} \leq \sqrt[n]{3^n} = 3$$

$$\frac{3^n}{2} > 2^n \quad n > 1$$

$a_n = 3$

$\{a_n\} = \{3, 3, \ldots\}$

$\lim_{n\to\infty} a_n = 3$

$$b_n = \sqrt[n]{3^n - \frac{3^n}{2}} = \sqrt[n]{\frac{1}{2} \cdot 3^n} =$$

$$= \sqrt[n]{\frac{1}{2}} \cdot \sqrt[n]{3^n} = \sqrt[n]{\frac{1}{2}} \cdot 3$$

$$\lim_{n\to\infty} 3 \cdot \underbrace{\sqrt[n]{\frac{1}{2}}}_{\to 1} = 3$$

ezért

$$\lim_{n\to\infty} \sqrt[n]{3^n - 2^n} = 3$$

Globális fgvtulajdonságok

- $f(x)$ periodikus, ha periodikus függvényekből származtatták.

  zérushelyek, értelmezési tartomány segít

- paritás $f(-x) = ?\ f(x)$ → $f(-x) = f(x)$ páros — grafikon y tengelyre szimmetrikus

  $f(-x) = -f(x)$ páratlan — grafikon origóra kp-an szimmetrikus

$f(x) = tgx + x$

$f(-x) = tg(-x) + (-x) = -(tgx + x)$ → páratlan

$f(x) = \sqrt{x}$ $\quad D_f: [0, \infty)$

↓

$\sqrt{-x}$ nincs ért.

$f(x) = \sin^2(x)$ $\quad f(-x) = (\sin(-x))^2 = (-\sin(x))^2 = \sin^2 x$ páros

$$f = \frac{x^2 - 2x}{x - 1} \qquad D_f = \mathbb{R} \setminus \{1\}$$

$\longrightarrow$ nem periodikus

$\longrightarrow$ nincs paritása

$$f = \frac{x^2 - 2x}{x^2 - 1} \qquad D_f = \mathbb{R} \setminus \{\pm 1\} \qquad \text{zh}: \{0, 2\}$$

$$f(-x) = \frac{(-x)^2 - 2(-x)}{(-x)^2 - 1} = \frac{x^2 + 2x}{x^2 - 1} \neq f(x)$$

$$\neq -f(x)$$

nincs paritás

Szakadásvizsgálat

$$f(x) = \frac{x^2 - x}{(x-1)(x+2)} \qquad D_f = \mathbb{R} \setminus \{1, -2\}$$

$zh: \{0, 1\} \rightarrow$ csak számláló!!

ha x_0 szakadás

$$\lim_{x \to x_0+} f(x) = A \qquad \lim_{x \to x_0-} f(x) = B$$

- $A = B \in \mathbb{R}$ megszüntethető
- $A, B \in \mathbb{R}$, $A \neq B$ ugrásos
- A, B nem véges $(\pm\infty)$ pólus
- $A \in \mathbb{R}$, $B = \pm\infty$ másodfajú

$x_0 = 1$

$$\lim_{x \to 1+} \frac{x^2 - x}{(x-1)(x+2)} = \lim_{x \to 1+} \frac{(x-1)x}{(x-1)(x+2)} = \lim_{x \to 1+} \frac{x}{x+2} = \frac{1}{3} = A$$

$$\lim_{x \to 1-} \frac{x^2 - x}{(x-1)(x+2)} = \lim_{x \to 1-} \frac{x}{x+2} = \frac{1}{3} = B$$

$A = B = \frac{1}{3} \in \mathbb{R}$

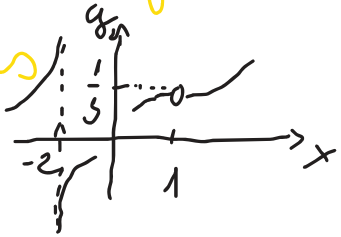
megszüntethető

$x_0 = -2$

$$\lim_{x \to -2+} \frac{x}{x+2} = \frac{-2}{0+} = -\infty = A$$

$$\lim_{x \to -2-} \frac{x}{x+2} = \frac{-2}{0-} = \infty = B$$

pólus



$$\lim_{x \to \infty} \left(\frac{x^2 - 1}{3 + x^2} \right)^{3x+2}$$

$$\lim_{x \to \infty} \left(1 + \frac{k}{\underbrace{f(x)}_{x \to \infty,\ \to \infty}} \right)^{\overset{x \to \infty}{f(x) \to \infty}} = e^{k}$$

$$\lim_{x \to \infty} \left(\left(\frac{x^2 + 3 - 4}{3 + x^2} \right)^{3+x^2} \right)^{\frac{3x+2}{3+x^2}} = \lim_{x \to \infty} \left(\underbrace{\left(1 + \frac{-4}{\underbrace{3+x^2}_{f(x)}} \right)^{\underbrace{3+x^2}_{f(x)}}}_{\to e^{-4}} \right)^{\overset{\to 0}{\frac{3x+2}{3+x^2}}} = \left(e^{-4}\right)^{0} = 1$$

$$\lim_{x \to \infty} \frac{3x+2}{3+x^2} = \text{"}\frac{\infty}{\infty}\text{"} = \lim_{x \to \infty} \frac{3 + \frac{2}{x}}{\underbrace{\frac{3}{x} + x}_{\to \infty}} = 0$$

$$\lim_{x \to \infty} \frac{3x^2+2}{3+x^2} = \text{"}\frac{\infty}{\infty}\text{"} = \lim_{x \to \infty} \frac{3 + \overset{\to 0}{\frac{2}{x^2}}}{\underbrace{\frac{3}{x^2}}_{\to 0} + 1} = \frac{3+0}{0+1} = 3$$

$$\lim_{x \to 0} \frac{\sin \cancel{x}\, f(x)}{\cancel{x}\, f(x)} = 1$$

$$\lim_{x \to 0} \frac{tg(2x)}{x} = \lim_{x \to 0} \frac{\sin 2x}{\cos 2x} \cdot \frac{1}{x} =$$

$$= \lim_{x \to 0} \underbrace{\frac{\sin 2x}{2x}}_{\to 1} \cdot 2 \cdot \frac{1}{\underset{\to 1}{\cos 2x}} = 1 \cdot 2 \cdot 1 = 2$$

$$2x \to 0$$

$$= \sin^2(5x)$$

$$\lim_{x \to 0} \frac{1 - \cos^2(5x)}{x^2} = \lim_{x \to 0} \Big(\underbrace{\frac{\sin 5x}{5x}}_{\to 1}\Big)^2 \cdot 25 = 1^2 \cdot 25 = 25$$

$$\sin^2 x + \cos^2 x = 1$$

$$f(g(x))\cdot g'(x)$$

$$\int \frac{1}{\operatorname{sh}^3 x}\cdot \operatorname{ch} x \, dx = \frac{-1}{2\cdot \operatorname{sh}^2 x} + C$$

Egyszerű helyettesítés

$$f(x) = \frac{1}{x^3} \qquad g(x) = \operatorname{sh} x$$

$$g'(x) = \operatorname{ch} x$$

$$\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + c = \frac{-1}{2x^2} + C$$

$$\int \sqrt{1-4x^2}\,dx = \int \underbrace{\sqrt{1-\sin^2 t}}_{\cos t} \cdot \frac{dx}{dt}\,dt = \int \cos t \cdot \frac{\cos t}{2}\,dt =$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos x = \sqrt{1-\sin^2 x}$$

$$4x^2 = \sin^2 t$$

$$\sin t = 2x$$

$$x = \frac{\sin t}{2} \qquad \frac{dx}{dt} = \frac{\cos t}{2}$$

$$= \frac{1}{2}\cdot \int \cos^2 t\,dt = \frac{1}{2}\cdot \int \frac{1+\cos 2t}{2}\,dt = \int \frac{1}{4}\,dt + \frac{1}{4}\int \cos 2t\,dt =$$

$$= \frac{1}{4}t + \frac{1}{4}\cdot\frac{\sin 2t}{2} + c = \frac{1}{4}\underset{\arcsin 2x}{t} + \frac{1}{8}\,2\cdot \underset{2x}{\sin t}\,\underset{\sqrt{1-4x^2}}{\cos t} + C$$

$$f(x) = \sqrt{1-4x^2} \qquad 1-4x^2 \geq 0$$

$$\frac{1}{4} \geqslant x^2 \qquad x \in \left[-\frac{1}{2}, \frac{1}{2}\right] = D_f$$

$$\downarrow$$

$x \in \left[0, \frac{1}{4}\right]$ $V = ?$ forgástest térfogat

$$V = \int_0^{\frac{1}{4}} \pi \cdot f^2(x)\,dx = \pi \cdot \int_0^{1/4} 1-4x^2\,dx =$$

$$= \pi \cdot \left[x - 4\frac{x^3}{3}\right]_0^{1/4} = \pi \cdot \left(\left(\frac{1}{4} - \cancel{4} \cdot \frac{1}{4^{\cancel{3}\,2}} \cdot \frac{1}{3}\right) - 0\right) =$$

$$= \pi \cdot \left(\frac{1}{4} - \frac{1}{4^2 \cdot 3}\right) = \pi \cdot \left(\frac{11}{4^2 \cdot 3}\right) = \pi\,\frac{11}{48}$$

Egyszerű hely $\int f(g(x)) \cdot g'(x)\, dx$

Parc. integ. $\int f(x) \cdot g(x)$

↑ polinom ← e^x, $\sin x$, $\cos x$, e^{2x}, $\sin 2x$ ...

Teljes hely

$\int f(g(x))\, dx$ → $t = g(x)$

$$\int \frac{1}{e^x + e^{2x}}$$

$t = e^x$