Matematika A1a -VIZSGA 2022.12.13. Megoldások

1. feladat

a) 3p

$$\lim_{n\to\infty} \frac{2^n-1}{5^n} = \lim_{n\to\infty} \underbrace{\left(\frac{2}{5}\right)^n}_{\to 0} - \underbrace{\left(\frac{1}{5}\right)^n}_{\to 0} = 0-0=0 \quad (2) \quad (1)$$

b) 4p

$$\lim_{n\to\infty} \sqrt[n]{n^2+2n-1} = 1$$

Rendőr-elv $\qquad \sqrt[n]{n} \xrightarrow{n\to\infty} 1 \qquad \sqrt[n]{3} \xrightarrow{n\to\infty} 1$

$$(1)\quad \left(\sqrt[n]{n}\right)^2 = \sqrt[n]{n^2} \le \sqrt[n]{n^2+2n-1} \le \sqrt[n]{3n^2} = \sqrt[n]{3}\cdot\left(\sqrt[n]{n}\right)^2 \quad (1)$$

$$2n-1>0 \qquad 2n-1<2n^2$$

$n\to\infty$: $1^2=1$ (1) $\longrightarrow$ $\underline{\underline{1}}$ $\longleftarrow$ (1) $1\cdot 1^2$ $\xleftarrow{n\to\infty}$

2.feladat

7p

$$x^2-4x+3=(x-1)(x-3) \qquad D_f=\mathbb{R}\setminus\{1,3\}$$ 1

$x_0=1$

$$\lim_{x\to 1-}\frac{1-x}{(x-1)(x-3)}=\lim_{x\to 1-}\frac{-1}{x-3}=\frac{-1}{-2}=\frac{1}{2}$$

$$\lim_{x\to 1+}\frac{1-x}{(x-1)(x-3)}=\lim_{x\to 1+}\frac{-1}{x-3}=\frac{-1}{-2}=\frac{1}{2}$$ 2

megszüntethető szakadás 1

$x_0=3$

$$\lim_{x\to 3-}\frac{1-x}{(x-1)(x-3)}=\lim_{x\to 3-}\frac{-1}{\underbrace{(x-3)}_{\to 0-}}=\frac{-1}{0-}=\infty$$

$$\lim_{x\to 3+}\frac{1-x}{(x-1)(x-3)}=\lim_{x\to 3+}\frac{-1}{\underbrace{x-3}_{\to 0+}}=\frac{-1}{0+}=-\infty$$ 2

pólus 1

3. feladat

5p

x: egy belépő ára $(x>0)$

$n\cdot\frac{3x^2}{100}$: ember nem megy be

Bevétel : $b(x) = n\cdot\underbrace{\left(1-\frac{3x^2}{100}\right)}_{\text{ennyien veszenek jegyet}}\cdot x = n\left(x-\frac{3x^3}{100}\right)$ 2

$b'(x) = n\cdot\left(1-\frac{9x^2}{100}\right)$ 1

$b'(x) = 0 \iff 1-\frac{9x^2}{100} = 0$

$x^2 = \frac{100}{9}$

$x = \frac{10}{3}$ 1

x	$0<x$	$x=\frac{10}{3}$	$\frac{10}{3}<x$
b'	$+$	0	$-$
b	$\nearrow$	MAX	$\searrow$

1

$f\left(\frac{10}{3}\right) = n\cdot\frac{20}{9}$

Ha a belépő ára $x=\frac{10}{3}$ akkor lesz a legtöbb bevétel.

4.feladat

10p

(1) $D_f = \mathbb{R}$

(2) zh: $x = 0$

(3) nincs spec. tul

1

vízsz. asz. $(y=0)$

(4) $\lim\limits_{x\to\infty} \frac{x^2}{e^x} = \text{"}\frac{\infty}{\infty}\text{"} \underset{L'H}{\rightsquigarrow} \lim\limits_{x\to\infty} \frac{2x}{e^x} = \text{"}\frac{\infty}{\infty}\text{"} \underset{L'H}{\rightsquigarrow} \lim\limits_{x\to\infty} \frac{2}{e^x} = 0$

$\lim\limits_{x\to -\infty} \frac{x^2}{e^x} = \frac{+\infty}{0+} = +\infty \cdot \infty = \infty$

3

(5) $f'(x) = \frac{2x \cdot e^x - x^2 \cdot e^x}{e^{2x}} = \frac{2x - x^2}{e^x} \overset{=0}{\iff} 2x - x^2 = 0$

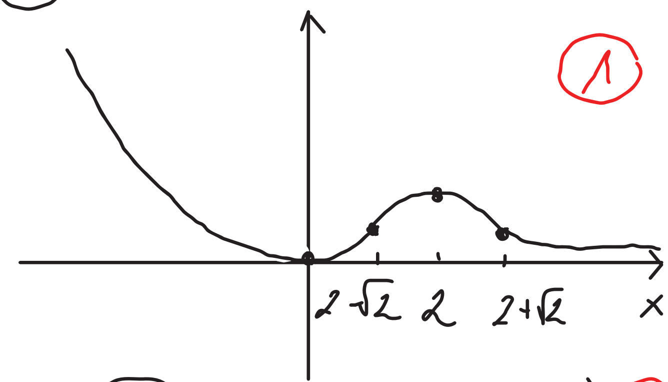
$x = 0$, $x = 2$

x	$x<0$	$x=0$	$0<x<2$	2	$2<x$
f'	$-$	0	$+$	0	$-$
f	$\searrow$	Min	$\nearrow$	Max	$\searrow$

$f(0) = 0$ $\quad f(2) = \frac{4}{e^2}$

2

(7)



1

(6) $f''(x) = \frac{(2-2x)e^x - (2x - x^2)e^x}{e^{2x}} =$

$= \frac{x^2 - 4x + 2}{e^x} \overset{=0}{\longrightarrow} x_{1,2} = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}$

2

x		$x=2-\sqrt{2}$		$x=2+\sqrt{2}$	
f''	$+$	0	$-$	0	$+$
f	$\smile$	Infl.	$\frown$	Infl.	$\smile$

(8) $R_f = [0, \infty)$ 1

5. feladat

5 + 4 p

a)

$$\int \frac{x^3+x^2}{x+2}\,dx = \int x^2 - x + 2\,dx + \int \frac{-4}{x+2}\,dx = \overbrace{\frac{x^3}{3} - \frac{x^2}{2} + 2x}^{1} \underbrace{- 4\ln|x+2| + C}_{2}$$

$$x^3 + x^2 : x+2 = x^2 - x + 2$$

$$\ominus\; x^3 + 2x^2$$

$$-x^2$$

$$\ominus\; -x^2 - 2x$$

$$+2x$$

$$\ominus\; 2x + 4$$

$$-4$$

2

b)

$$\int \underset{f'}{x} \cdot \underset{g}{\ln(x)}\,dx = \frac{x^2}{2}\cdot\ln(x) - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx = \frac{x^2}{2}\cdot\ln(x) - \frac{x^2}{4} + C$$

2 2

6. feladat

7p

$$f(x)=\sqrt{2x} \qquad x\in[0,2] \qquad f'(x)=\frac{1}{2}\cdot\frac{1}{\sqrt{2x}}\cdot 2=\frac{1}{\sqrt{2x}} \quad 1$$

Felszín

$$A=\int_0^2 2\pi\cdot f(x)\cdot\sqrt{1+f'(x)^2}\,dx = 2\pi\int_0^2 \sqrt{2x}\cdot\sqrt{1+\frac{1}{2x}}\,dx = \quad 2$$

$$=\cancel{2}\pi\int_0^2 \underset{\substack{\uparrow\\ f(g(x))}}{\sqrt{2x+1}}\cdot \underset{\substack{\uparrow\\ g'(x)}}{2}\cdot\frac{\cancel{1}}{\cancel{2}}\,dx = \pi\cdot\left[\frac{(2x+1)^{3/2}}{3/2}\right]_0^2 = \quad 2$$

$$=\frac{2}{3}\pi\cdot\left(\sqrt{(2\cdot 2+1)^3}-\sqrt{(2\cdot 0+1)^3}\right)=\frac{2}{3}\pi\cdot\left(5\cdot\sqrt{5}-1\right) \quad 2$$