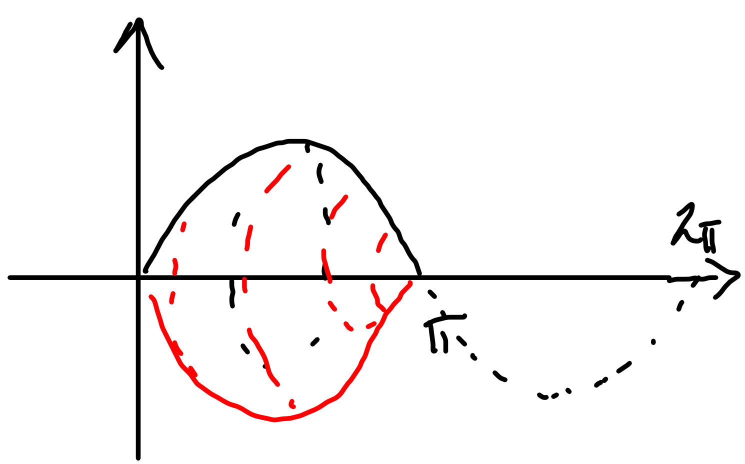
Konzultáció P0 2023.12.12.

Felszín forgástestre $\quad f(x) \geq 0 \quad x \in [a,b]$

$$F_f^{[a,b]} = 2\pi \cdot \int_a^b f(x) \cdot \sqrt{1+(f'(x))^2}\, dx$$

$\sin(x) \quad x \in [0,\pi]$



$f(x) = \sin(x)$

$f'(x) = (\sin(x))' = \cos(x)$

$(f')^2 = \cos^2(x)$

$$F = 2\pi \int_0^\pi \sin(x) \cdot \sqrt{1+\cos^2(x)}\, dx$$

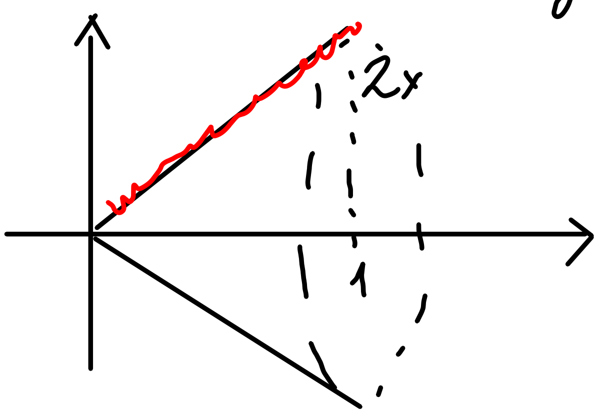
$$2\pi\int_0^\pi \sin(x)\cdot\sqrt{1+\cos^2(x)}\,dx$$

$$\cos(x)=\operatorname{sh}(t)$$

$$\sqrt{1+\cos^2(x)}=\sqrt{1+\operatorname{sh}^2(t)}=\sqrt{\operatorname{ch}^2(t)}=\operatorname{ch}(t)$$

Kúp felszíne

$$f(x)=2x \qquad x\in[0,1]$$



$$F=2\pi\cdot\int_0^1 f(x)\cdot\sqrt{1+(f'(x))^2}\,dx = *$$

$$f(x)=2x$$
$$f'(x)=2$$
$$(f'(x))^2=4$$

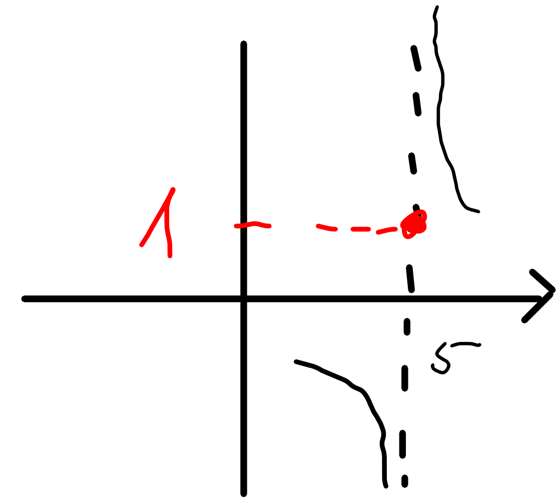
$$*=2\pi\int_0^1 2x\cdot\sqrt{1+4}\,dx = 2\pi\cdot\sqrt{5}\cdot\int_0^1 2x\,dx = 4\sqrt{5}\pi\cdot\left[\frac{x^2}{2}\right]_0^1 =$$

$$=4\sqrt{5}\cdot\pi\cdot\left(\frac{1^2}{2}-\frac{0^2}{2}\right)=2\sqrt{5}\cdot\pi$$

Szakadási hely:

a) $f(x) = \dfrac{3x-2}{(x-5)\,x}$ $\qquad D_f: \mathbb{R} \setminus \{5, 0 \dots\}$

$\qquad 5^-$



$$f(x) = \begin{cases} \dfrac{3x-2}{x-5} & x \neq 5 \\ 1 & x = 5 \end{cases}$$

$$\lim_{x \to 5+} \frac{\overset{\to 13}{3x-2}}{\underset{\to 0+}{x-5}} = \frac{13}{0+} = \infty$$

$$\lim_{x \to 5-} \frac{\overset{\to 13}{3x-2}}{\underset{\to 0-}{x-5}} = \frac{13}{0-} = -\infty$$

lényeges szakadás

PÓLUS

b)

$$f(x) = \frac{1}{e^{1/x^2}}$$

$\frac{1}{x^2} \to x = 0$-nál nincs értelmezve

$e^x > 0 \;\ldots$

$D_f: \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$

$$\lim_{x \to 0+} \frac{1}{e^{1/x^2}} = \frac{1}{\infty} = 0$$

$1/x^2 \to \infty$, $e^{1/x^2} \to \infty$

$x \to 0$

$x^2 \to 0+$

$1/x^2 \to \infty$

$e^t \xrightarrow[t \to \infty]{} \infty$

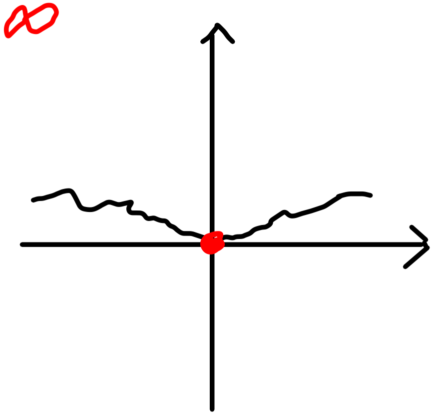
$$\lim_{x \to 0-} \frac{1}{e^{1/x^2}} = \frac{1}{\infty} = 0$$

$1/x^2 \to \infty$, $e^{1/x^2} \to \infty$

$x \to 0-$

$x^2 \to 0+$

$1/x^2 \to \infty$



megszüntethető szakadás $x = 0$-ban

$$\int \frac{1}{\underbrace{x^2-2x+5}} dx = \int \frac{1}{(x-1)^2+4} dx = \frac{1}{4} \cdot \int \frac{1}{\left(\frac{x-1}{4}\right)^2+1} dx = *$$

szorzatra bontani!

$D = 4 - 4 \cdot 5 < 0$

nem tudunk

Teljes négyzet

$$\int \frac{1}{x^2+1} dx = \operatorname{arctg}(x) + C$$

$$* = \frac{1}{4} \cdot 2 \cdot \int \underbrace{\frac{1}{\left(\frac{x-1}{2}\right)^2+1}}_{f(g(x))} \cdot \underbrace{\frac{1}{2}}_{g'(x)} dx = \frac{1}{4} \cdot 2 \cdot \underbrace{\operatorname{arctg}\left(\frac{x-1}{2}\right)}_{F(g(x))} + C = \frac{1}{2} \cdot \operatorname{arctg}\left(\frac{x-1}{2}\right) + C$$

$g(x) = \frac{x-1}{2}$ $\qquad g'(x) = \left(\frac{x-1}{2}\right)' = \frac{1}{2}$

$f(x) = \frac{1}{x^2+1}$ $\qquad F(x) = \int f \, dx = \operatorname{arctg}(x) + C$

$$\int \frac{\cos(x)}{\sqrt{\sin(x)}}\,dx = \int \frac{1}{\sqrt{\sin(x)}} \cdot \cos(x)\,dx = 2\cdot\sqrt{\sin(x)} + C$$

$$F(g(x)) + C$$

$$\underbrace{\frac{\cos(x)}{\sqrt{\sin(x)}}}_{f(g(x))\,\cdot\, g'(x)}$$

$$f(x) = \frac{1}{\sqrt{x}} \qquad g(x) = \sin(x)$$

$$g'(x) = \cos(x)$$

$$F(x) = \int f(x)\,dx = \int \frac{1}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\,dx =$$

$$= \frac{x^{1/2}}{1/2} + C = 2\cdot\sqrt{x} + C$$