Konzultáció P0 kurzus 2024.01.08.

Helyettesítéses integrálás

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{e^t}{t}\cdot dx = \int \frac{e^t}{t}\underbrace{\frac{dx}{dt}}_{\dot{x}(t)=(t^2)'=2t}\,dt = \int \frac{e^t}{t}\cdot 2t\,dt =$$

$\sqrt{x} = t$ $\quad (x \geq 0)$

$x = t^2$

$$= \int 2e^t\,dt = 2e^t + c = 2e^{\sqrt{x}} + c$$

$$\left(2\cdot e^{\sqrt{x}}\right)' = 2\cdot e^{\sqrt{x}}\cdot\left(\sqrt{x}\right)' = \cancel{2}\cdot e^{\sqrt{x}}\cdot\frac{1}{\cancel{2}}\cdot\frac{1}{\sqrt{x}} = \frac{e^{\sqrt{x}}}{\sqrt{x}}$$

Határozott integrál

$$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

$$F'(x) = f(x)$$

$$\int_e^{e^2} \frac{\ln(\ln(x))}{x}\,dx = \int_1^2 \frac{\ln(t)}{\cancel{e^t}} \cdot \cancel{e^t} \cdot dt = \int_1^2 \ln(t)\,dt$$

$$\begin{cases} x_a = e \\ \ln(e) = t_a \\ 1 = t_a \end{cases}$$

$\ln(x) = t$

$e^{\ln(x)} = e^t$

$x = e^t$

$$\frac{dx}{dt} = \left(e^t\right)' = e^t$$

$$\begin{cases} x_b = e^2 \\ \ln(e^2) = t_b \\ 2 = t_b \end{cases}$$

parc. int

$$\int \underbrace{1 \cdot \ln(t)}_{f' \cdot g}\,dt = \underbrace{t \cdot \ln(t)}_{f \cdot g} - \underbrace{\int t \cdot \frac{1}{t}\,dt}_{\int f \cdot g'} =$$

$$= t \cdot \ln(t) - t + c$$

$f' = 1 \quad f = t$

$g = \ln(t) \quad g' = \frac{1}{t}$

$$\int_1^2 \ln(t)\,dt = \left[t \cdot \ln(t) - t\right]_1^2 =$$

$$= (2 \cdot \ln(2) - 2) - (1 \cdot \underbrace{\ln(1)}_{=0} - 1) =$$

$$= 2 \cdot \ln(2) - 2 - 0 + 1 = \underline{\underline{2 \cdot \ln(2) - 1}}$$

$$\int_2^3 \frac{5}{(x-1)^3}\,dx = \left[-\frac{5}{2} \cdot \frac{1}{(x-1)^2}\right]_2^3 = \left(\frac{-5}{2} \cdot \frac{1}{(3-1)^2}\right) - \left(\frac{-5}{2} \cdot \frac{1}{(2-1)^2}\right) = *$$

$$(F(x)) = \int \frac{5}{(x-1)^3}\,dx = 5 \cdot \int \frac{1}{(\underbrace{x-1})^3} \cdot 1\,dx = 5 \cdot \frac{(x-1)^{-2}}{-2} + C$$

$$f = x-1 \quad f' = 1$$

$$g = x^{-3} \quad \int g\,dx = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + C$$

$$* = -\frac{5}{8} + \frac{5}{2} = \frac{20}{8} - \frac{5}{8} = \frac{15}{8}$$

$$\int \frac{1}{x^2+4x+3}\,dx$$

1. Polinomosztás $\Rightarrow$ nem kell
2. x^2+4x+3 szorzatalak
   $x^2+4x+3=(x+1)(x+3)$
3. Parc. tört.

$$(x+1)^1 \longrightarrow \frac{A}{x+1}$$

$$(x+3)^1 \longrightarrow \frac{B}{x+3}$$

$$\frac{1}{x^2+4x+3} = \frac{A}{x+1} + \frac{B}{x+3} = \frac{A(x+3)+B(x+1)}{(x+1)(x+3)}$$

$$1 = A(x+3) + B(x+1)$$

$$\boxed{0}x + \boxed{1} = Ax + 3A + Bx + B = \underbrace{\boxed{(A+B)}}_{=0}x + \underbrace{\boxed{(3A+B)}}_{=1}$$

① $A + B = 0$

② $3A + B = 1$

② − ① $\Rightarrow 2A + 0 = 1$

$A = \frac{1}{2}$ ① $\Rightarrow B = -\frac{1}{2}$

$$\int \frac{1}{x^2+4x+3}\,dx = \int \frac{A}{x+1} + \frac{B}{x+3}\,dx = \int \frac{1/2}{x+1} + \frac{-1/2}{x+3}\,dx =$$

$$= \frac{1}{2} \cdot \int \frac{1}{x+1}\,dx - \frac{1}{2} \cdot \int \frac{1}{x+3}\,dx = \frac{1}{2} \cdot \ln|x+1| - \frac{1}{2} \cdot \ln|x+3| + c$$

$\cdot 1 = f'$

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

$f = x+1$

$x+3$

$$\int \frac{x^4+1}{x^2\cdot(x-1)}\,dx = \int \frac{x^4+1}{x^3-x^2}\,dx = *$$

① Polinomosztás x^4+1 fokszám $=4 > x^3-x^2$ fokszáma $=3$

$$x^4+1 : x^3-x^2 = x+1$$

$$\begin{array}{l} \ominus\; x^4 - x^3 \\ \hline 0 + x^3 + 1 \\ \ominus\; x^3 - x^2 \\ \hline 0 + \boxed{x^2+1} \end{array}$$

$\cdot x$, $\cdot 1$

$$(x^4+1) = (x+1)(x^3-x^2) + x^2+1$$

$$\int \frac{(x+1)\cancel{(x^3-x^2)}}{\cancel{x^3-x^2}} + \frac{x^2+1}{x^3-x^2}\,dx = \int x+1\,dx + \int \frac{x^2+1}{x^3-x^2}\,dx =$$

$$= \frac{x^2}{2} + x + \int \frac{x^2+1}{x^2(x-1)}\,dx = *$$

② szorzatalak ok $(x^2 \cdot (x-1))$

③ parc. törtek

$$x^2 = (x-0)^2 \longrightarrow \frac{A}{x} + \frac{B}{x^2}$$

$$(x-1)^1 \longrightarrow \frac{C}{x-1}$$

$$\frac{x^2+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} = \frac{Ax(x-1)+B(x-1)+Cx^2}{x^2(x-1)}$$

$$x^2+1 = A(x^2-x) + B(x-1) + Cx^2$$

$$x^2 \overset{+0\cdot x}{+} 1 = Ax^2 - Ax + Bx - B + Cx^2$$

$$\left.\begin{aligned} 1 &= A + C \\ 0 &= -A + B \\ 1 &= -B \end{aligned}\right\} \quad \left.\begin{aligned} 1 &= -1 + C \to C = 2 \\ 0 &= -A + -1 \to A = -1 \\ 1 &= -B \to B = -1 \end{aligned}\right\}$$

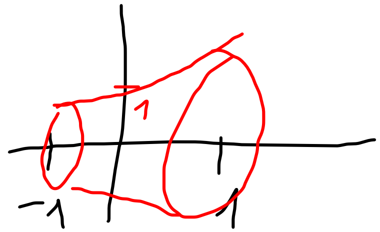
$$\int \frac{x^2+1}{x^2(x-1)}\,dx = \int \frac{-1}{x} + \frac{-1}{x^2} + \frac{2}{x-1}\,dx =$$

$$= -\int \frac{1}{x}\,dx - \int \frac{1}{x^2}\,dx + 2\int \frac{1}{x-1}\,dx =$$

$$= -\ln|x| - \frac{x^{-1}}{-1} + 2\ln|x-1| + c$$

$f(x) = e^{x/4}$ $\quad x \in [\overset{a}{-1}, \overset{b}{1}]$ $\rightarrow$ térfogat

$$V = \int_a^b f^2(x) \cdot \pi \, dx =$$



$$= \pi \cdot \int_{-1}^{1} \left(e^{x/4}\right)^2 dx = \pi \cdot \int_{-1}^{1} e^{\frac{x}{2}} dx =$$

f'

$g(f(x))$

$\overset{\shortparallel}{e^x} \qquad \searrow \frac{x}{2}$

$\int g \, dx = \int e^x dx = e^x + C$

$f = \frac{x}{2} \qquad f'(x) = \frac{1}{2}$

$$= 2 \cdot \pi \cdot \int_{-1}^{1} \overbrace{e^{\frac{x}{2}} \cdot \frac{1}{2}}^{g(f(x)) \cdot f'(x)} dx = 2\pi \left[e^{\frac{x}{2}} \right]_{-1}^{1} =$$

$$= 2\pi \cdot \left(\left(e^{1/2} \right) - \left(e^{-1/2} \right) \right) = 2\pi \left(\sqrt{e} - \frac{1}{\sqrt{e}} \right)$$