Konzultáció P0 kurzus   2024.01.15.

$$\lim_{n\to\infty} \frac{\overbrace{2^n-1}^{\to\infty}}{\underbrace{5^n}_{\to\infty}} = "\frac{\infty}{\infty}" = \lim_{n\to\infty} \frac{2^n}{5^n} - \frac{1}{5^n} =$$

$$= \lim_{n\to\infty} \underbrace{\left(\frac{2}{5}\right)^n}_{\to 0} - \underbrace{\left(\frac{1}{5}\right)^n}_{\to 0} = 0 - 0 = 0$$

Szöveges szélsőérték

Fenyőfák árusítása 50 db ültethető

x m-es fa 6 petákot ér

Kivágáskor a haszon (abban az évben)

$50 \cdot 6 \cdot x$

Az évekre szétoszló haszon $\boxed{h(x) = \dfrac{50 \cdot 6 \cdot x}{4+x^2}} \to \text{Max}$

$h(x) = \dfrac{50 \cdot 6x}{4+x^2}$ $\qquad h'(x) \rightarrow$ hol 0? $\rightarrow$ lok. szélső-érték

$D_h = ?$

+ igazoljuk, hogy D_h-n globális max.-ot találtunk

① $D_h: \; x > 0 \quad (x \in (0, \infty))$

② $h'(x) = 0$?

$$h'(x) = \left(\frac{300x}{4+x^2}\right)' = \frac{300 \cdot (4+x^2) - 300x \cdot 2x}{(4+x^2)^2} \stackrel{?}{=} 0$$

$$30\cancel{0} \cdot (4+x^2) - 30\cancel{0} \cdot 2 \cdot x^2 \stackrel{?}{=} 0$$

$$4 + x^2 - 2x^2 = 0$$

$$4 - x^2 = 0$$

$$x^2 = 4 \quad \begin{matrix} x = -2 \;\; \text{↯} \;\; \notin D_h \\ x = 2 \end{matrix}$$

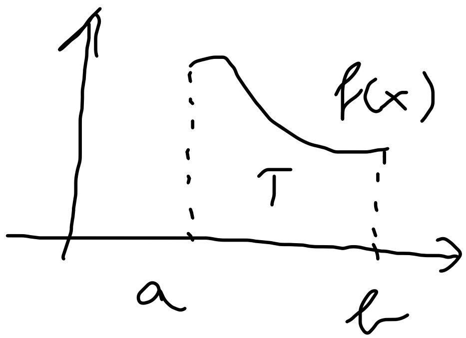
③

x	$(0,2)$	2	$(2,\infty)$
h'	$\frac{+\cdot+}{+} = +$	0	$\frac{+\cdot-}{+} = -$
h	$\nearrow$	MAX	$\searrow$

$$h'(x) = \frac{300(4-x^2)}{(4+x^2)^2} > 0$$

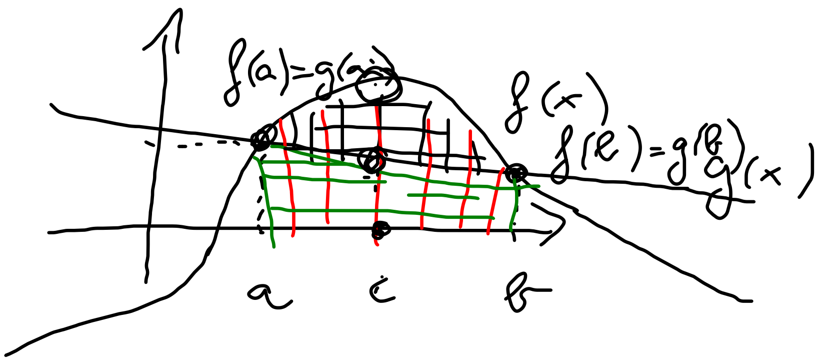
2 értékén lesz a legnagyobb a haszon.

Területszámítás integrállal



$$T = \int_a^b f(x)\,dx = \left[F(x)\right]_a^b$$

$$F'(x) = f(x)$$



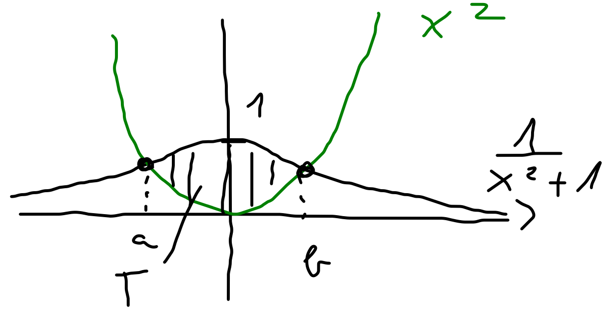
$a, b = ?$ $\quad f(x) = g(x)$

$$T_f = \int_a^b f(x)\,dx \qquad T_g = \int_a^b g(x)\,dx$$

$$T = T_f - T_g = \int_a^b f(x) - g(x)\,dx$$

Konkrét példa:

$f(x) = x^2 \qquad g(x) = \frac{1}{x^2+1}$



$$T = ? = \int_a^b g(x) - f(x)\, dx$$

$a, b = ?$ $\qquad x^2 = \frac{1}{x^2+1} \rightarrow$

$$x^2(x^2+1) = 1$$
$$x^4 + x^2 = 1$$
$$\underbrace{x^4 + x^2 - 1}_{z = x^2} = 0$$
$$z^2 + z - 1 = 0$$
$$z_{1,2} = \frac{-1 \pm \sqrt{1 - 4\cdot(-1)}}{2} = -\frac{1}{2} \pm \frac{\sqrt{5}}{2} \rightarrow 1$$

$x^2 > 0 \quad (z > 0)$

$z = -\frac{1}{2} + \frac{\sqrt{5}}{2} = x^2$
> 0

$a = -\sqrt{-\frac{1}{2} + \frac{\sqrt{5}}{2}} \qquad b = \sqrt{-\frac{1}{2} + \frac{\sqrt{5}}{2}}$

$$T = \int_{-\sqrt{-\frac{1}{2}+\frac{\sqrt{5}}{2}}}^{\sqrt{-\frac{1}{2}+\frac{\sqrt{5}}{2}}} \frac{1}{x^2+1} - x^2 \, dx = \left[\operatorname{arctg}(x) - \frac{x^3}{3} \right]_{-\sqrt{-\frac{1}{2}+\frac{\sqrt{5}}{2}}}^{\sqrt{-\frac{1}{2}+\frac{\sqrt{5}}{2}}} =$$

$$= \left(\operatorname{arctg}\left(\overset{b}{\sqrt{-\frac{1}{2}+\frac{\sqrt{5}}{2}}} \right) - \frac{\left(\overset{b}{\sqrt{-\frac{1}{2}+\frac{\sqrt{5}}{2}}} \right)^3}{3} \right) - \left(\operatorname{arctg}\left(-\overset{a}{\sqrt{\dots}} \right) - \frac{\left(-\overset{a}{\sqrt{\dots}} \right)^3}{3} \right)$$

arctg páratlan

x^3 is!

$$= 2 \cdot \operatorname{arctg}\left(\sqrt{-\frac{1}{2}+\frac{\sqrt{5}}{2}} \right) - \frac{2}{3} \cdot \left(\sqrt{-\frac{1}{2}+\frac{\sqrt{5}}{2}} \right)^3$$

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$$\int \frac{1}{x^2-4}\,dx = \int \frac{1}{(x-2)(x+2)}\,dx =$$

$$= \int \frac{A}{x-2} + \frac{B}{x+2}\,dx = \dots\ *$$

$$\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{A(x+2)+B(x-2)}{(x-2)(x+2)}$$

$$0 \cdot x + 1 = Ax + 2A + Bx - 2B$$

$$\left.\begin{aligned} 0 &= A + B \\ 1 &= 2A - 2B \end{aligned}\right\} \quad 2 + 1 \cdot 2 \Rightarrow$$

$$1 + 2 \cdot 0 = 1 = 4A + 0B$$

$$A = \frac{1}{4} \quad B = \frac{-1}{4}$$

$$* = \int \frac{\frac{1}{4}}{x-2} + \frac{-\frac{1}{4}}{x+2}\, dx =$$

$$= \frac{1}{4} \cdot \int \frac{1}{x-2}\, dx - \frac{1}{4} \int \frac{1}{x+2}\, dx = **$$

$$\int \frac{1}{x+a}\, dx = \ln|x+a| + C \qquad \int \frac{1}{x+a}\, dx = \ln|x+a| + c$$

$$** = \frac{1}{4} \cdot \ln|x-2| - \frac{1}{4} \cdot \ln|x+2| + C =$$

$$= \ln\left|\frac{x-2}{x+2}\right|^{\frac{1}{4}} + c$$

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$$\lim_{n\to\infty} \frac{\overset{\to\infty}{\sqrt{n}} - \overset{\to\infty}{2n}}{\underset{\to\infty}{\sqrt{n^2+4}}} = "\frac{\infty - \infty}{\infty}" = "\frac{-\infty}{\infty}" =$$

$$= \lim_{n\to\infty} \frac{\sqrt{n} - 2n}{\sqrt{n^2+4}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{\sqrt{n}}{n} - \frac{2\cancel{n}}{\cancel{n}}}{\sqrt{\frac{n^2}{n^2} + \frac{4}{n^2}}} =$$

$$= \lim_{n\to\infty} \frac{\overset{\to 0}{\frac{1}{\sqrt{n}}} \overset{\to -2}{- 2}}{\sqrt{\underset{\to 1}{1} + \underset{\to 0}{\frac{4}{n^2}}}} = \frac{0-2}{\sqrt{1+0}} = -2$$