Konzultáció P0 kurzus 2024.01.22.

Határértékek

$$\lim_{x\to 0} \frac{\sin(2x)}{\pi \cdot x} = ?$$

$$\lim_{x\to 0} \frac{\sin(2x)}{3 \cdot x} = \lim_{x\to 0} \frac{\sin(2x)}{2x} \cdot \frac{1}{3} \cdot 2 = 1 \cdot \frac{1}{3} \cdot 2 = \frac{2}{3}$$

$\frac{\sin(2x)}{2x} \to 1$

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1 = "\frac{0}{0}" \left(= \lim_{x\to 0} \frac{x}{\sin(x)} = 1 \right)$$

$$\lim_{x\to 0} \frac{1}{\pi} \cdot \frac{\sin(2x)}{2x} \cdot 2 = \frac{1}{\pi} \cdot 1 \cdot 2 = \frac{2}{\pi}$$

$$\lim_{x\to ?} \frac{\sin(\ldots)}{\ldots} = 1$$

$x \to 0$, $\ldots \to 0$

$$\lim_{x\to 1} \frac{\sin(x-1)}{x-1} = 1$$

$x-1 \to 0$

$$\lim_{n\to\infty} \underbrace{\left(1+\frac{2}{n}\right)}_{\to 1}^{\overbrace{n}^{\to\infty}} = e^{k} \qquad \lim_{n\to\infty}\left(1+\frac{2}{\underbrace{\cdots}_{\to\infty}}\right)^{\overbrace{\cdots}^{\to\infty}} = e^{k}$$

$$\lim_{n\to\infty}\left(1-\frac{2}{\underbrace{2n+1}_{\to\infty}}\right)^{\overbrace{2n+1}^{\to\infty}} = e^{-2}$$

$$\lim_{n\to\infty}\left(\frac{2n-2}{2n+1}\right)^{2n+1} = \lim_{n\to\infty}\left(\frac{2n+1-3}{2n+1}\right)^{2n+1} =$$

$$= \lim_{n\to\infty}\left(1-\frac{3}{2n+1}\right)^{2n+1} = e^{-3} \qquad a^{b}\cdot a^{c} = a^{b+c}$$

$$\lim_{n\to\infty}\left(\frac{2n-2}{2n+1}\right)^{2n+2} = \lim_{n\to\infty}\left(\frac{2n-2}{2n+1}\right)^{2n+1}\cdot\left(\frac{2n-2}{2n+1}\right)^{1} =$$

$$= \lim_{n\to\infty}\left(1-\frac{3}{2n+1}\right)^{2n+1}\cdot\frac{2n-2}{2n+1} = \frac{1-\overbrace{\frac{2}{2n}}^{\to 0}}{1+\underbrace{\frac{1}{2n}}_{\to 0}} = e^{-3}\cdot 1 = e^{-3}$$

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^{x}=e^{k}$$

$$\lim_{x\to 0^+}\left(1+\frac{k}{1/x}\right)^{1/x}=e^{k}\qquad \lim_{x\to\infty}\left(1+\frac{k}{\sqrt{x}}\right)^{\sqrt{x}}=e^{k}$$

$$\lim_{x\to\infty}\left(\frac{x^2+\pi}{x^2}\right)^{x^2+4}=\lim_{x\to\infty}\underbrace{\left(1+\frac{\pi}{x^2}\right)^{x^2}}_{\to e^{\pi}}\cdot\underbrace{\left(1+\frac{\pi}{x^2}\right)^{4}}_{\to 1}=e^{\pi}\cdot 1^{4}=e^{\pi}$$

Szakadás felismerése

$$f(x) = \frac{x-1}{x^2-x} \qquad D_f: \mathbb{R} \setminus \{0, 1\}$$

(olyan nincs $f(0)$, $f(1)$ ....)

$$x^2 - x \neq 0$$
$$x(x-1) \neq 0$$
$$x_1 \neq 0 \qquad x_2 \neq 1$$

$x_0 = 0$

$$\lim_{x \to 0+} \frac{\overset{\to -1}{x-1}}{\underset{\to 0}{x^2-x}} = \lim_{x \to 0+} \frac{\cancel{x-1}^{\,\to -1}}{\underset{\to 0+}{x} \cdot \underset{\to -1}{\cancel{(x-1)}}} = \lim_{x \to 0+} \frac{1}{x} = \frac{1}{0+} = \infty$$

$$\lim_{x \to 0-} \frac{x-1}{x^2-x} = \lim_{x \to 0-} \frac{1}{x} = \frac{1}{0-} = -\infty$$

$\Rightarrow$ pólus

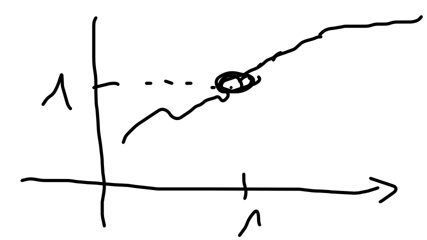
lényeges szakadás

$x_0 = 1$

$$\lim_{x \to 1+} \frac{\overset{\to 0+}{\cancel{x-1}}}{\underset{\to 1}{x} \cdot \underset{\to 0+}{\cancel{(x-1)}}} = \text{"}\frac{0}{1 \cdot 0}\text{"} = \lim_{x \to 1+} \frac{1}{\underset{\to 1+}{x}} = 1$$

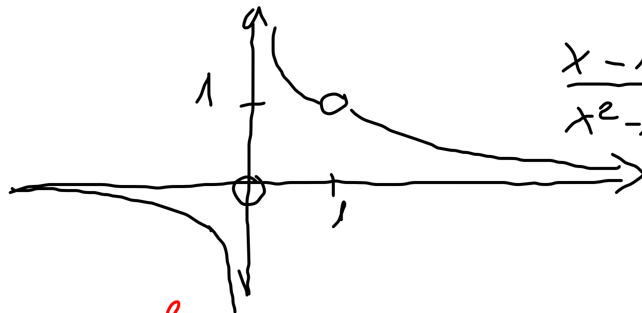
$$\lim_{x \to 1-} \frac{x-1}{x \cdot (x-1)} = \text{"}\frac{0}{0}\text{"} = \lim_{x \to 1-} \frac{1}{\underset{\to 1-}{x}} = 1$$

megszüntethető szakadás



$$\lim_{x\to\infty} \frac{\overset{\to\infty}{x-1}}{\underset{\searrow\,\infty-\infty}{x^2-x}} = "\frac{\infty}{\infty-\infty}" = \lim_{x\to\infty} \frac{\overset{0\leftarrow}{\frac{1}{x}} - \overset{\to 0}{\frac{1}{x^2}}}{1-\underset{\to 0}{\frac{1}{x}}} = \frac{0-0}{1-0} = \frac{0}{1} = 0$$

$$\lim_{x\to-\infty} = 0$$



$$\frac{x-1}{x^2-x} \sim \frac{1}{x}$$

Zay: bezorzás

$$\lim_{n\to\infty} \overbrace{\underbrace{\sqrt{n^4+2n}}_{\downarrow\,\infty}}^{a} - \overbrace{\underbrace{n^2}_{\downarrow\,\infty}}^{b} = "\infty-\infty" = \qquad (a-b)\cdot(a+b) = a^2-b^2$$

$$= \lim_{n\to\infty} \left(\sqrt{n^4+2n} - n^2\right)\cdot\frac{\left(\sqrt{n^4+2n}+n^2\right)}{\sqrt{n^4+2n}+n^2} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n^4}+2n-\cancel{n^4}}{\sqrt{n^4+2n}+n^2} = \lim_{n\to\infty} \frac{\overset{\to\infty}{2n}}{\underbrace{\sqrt{n^4+2n}}_{\searrow\,\infty} + \underbrace{n^2}_{\searrow\,\infty}} = "\frac{\infty}{\infty}" \frac{1/n^2}{1/n^2} =$$

$$\lim_{n \to \infty} \frac{\frac{2}{n} \to 0}{\sqrt{\underset{\downarrow\, 1}{1} + \underset{\downarrow\, 0}{\frac{2}{n^3}}} + \underset{\searrow\, 1}{1}} = \frac{0}{\sqrt{1+0}+1} = \frac{0}{2} = 0$$

$$\lim_{n \to \infty} \frac{\sqrt{n^4 + 2n} - n^2}{n^3} = \frac{\text{"}\infty - \infty\text{"}}{\infty} =$$

$$\lim_{n \to \infty} \frac{\left(\sqrt{n^4+2n} - n^2\right) : n^3}{n^3 \quad : n^3} = \lim_{n \to \infty} \frac{\sqrt{\frac{n^4}{n^6} + \frac{2n}{n^6}} - \frac{n^2}{n^3}}{1} =$$

$$= \lim_{n \to \infty} \frac{\sqrt{\frac{1}{n^2} + \frac{2}{n^5}} - \frac{1}{n}}{1} = \frac{\sqrt{0+0} - 0}{1} = \frac{0}{1} = 0$$

Primitív fgvek keresése (integráltáblázat)

$$\int f(g(x)) \cdot g'(x)\, dx = F(g(x)) + C$$

$\hookrightarrow F'(x) = f$

$$\int \underbrace{\sin(x^2)} \cdot 2x\, dx = -\cos(x^2) + C$$

$f = \sin \quad \to F = \int \sin(x)\,dx = -\cos(x) + C$

$g = x^2$

(táblázat)

$$\int \frac{\ln(\sqrt{x})}{\sqrt{x}}\, dx = 2 \cdot \int \ln(\sqrt{x}) \cdot \underbrace{\frac{1}{2\sqrt{x}}}_{g'}\, dx = 2\left(\sqrt{x} \cdot \ln(\sqrt{x}) - \sqrt{x}\right) + C$$

$f = \ln \quad g = \sqrt{x}$

$g' = \frac{1}{2\sqrt{x}}$

$F(x) = \int \ln(x)\, dx = x \cdot \ln(x) - x + C$

$$\int f'(x) \cdot g(x)\, dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x)\, dx$$

$$\int x \cdot e^{2x}\, dx = \frac{e^{2x}}{2} \cdot x - \int \frac{e^{2x}}{2} \cdot 1\, dx =$$

$g(x) = x \quad g'(x) = 1$

$f'(x) = e^{2x} \quad f(x) = \frac{e^{2x}}{2}$

$\int e^x = e^x + c$

$\frac{1}{2} \cdot \int e^{2x} \cdot 2 = \frac{e^{2x}}{2} + c$

$$= \frac{e^{2x}}{2} \cdot x - \frac{1}{2} \cdot \frac{1}{2} \cdot \int e^{2x} \cdot 2\, dx =$$

$$= \frac{e^{2x}}{2} \cdot x - \frac{1}{4} \cdot e^{2x} + c =$$

$$= e^{2x} \cdot \left(\frac{x}{2} - \frac{1}{4} \right) + c$$

$$\int \frac{2x+2}{x^2+1}\, dx = \int \frac{1}{x^2+1} \cdot 2x + \frac{1}{x^2+1} \cdot 2\, dx =$$

$$\int \frac{1}{x^2+1} \cdot \underbrace{2x}_{g'}\, dx = \ln|x^2+1| + c$$

$F(g(x))$

$f = \frac{1}{x} \quad g = x^2+1$

$F = \int \frac{1}{x} dx = \ln|x| + c \quad g' = 2x$

$$\int \frac{1}{x^2+1} \cdot 2 \, dx = 2 \cdot \int \frac{1}{x^2+1} \, dx = 2 \cdot \operatorname{arctg}(x) + C$$

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$$\int_1^2 \frac{x^3+2x}{x+1} \, dx =$$

$$
\begin{array}{l}
x^3+2x : x+1 = x^2-x+3 \\
x^3+x^2 \\
\hline
-x^2+2x \\
-x^2-x \\
\hline
3x \\
3x+3 \\
\hline
-3
\end{array}
$$

$$= \int_1^2 x^2-x+3-\frac{3}{x+1} \, dx =$$

$$\int -\frac{3}{x+1} \, dx = -3 \cdot \int \frac{1}{x+1} \cdot 1 \, dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} + 3x - 3 \cdot \ln|x+1| \right]_1^2 =$$

$$= \left(\frac{2^3}{3} - \frac{2^2}{2} + 3 \cdot 2 - 3 \cdot \ln|2+1| \right) - \left(\frac{1}{3} - \frac{1}{2} + 3 - 3 \cdot \ln|2| \right)$$