Konzultáció P0 2024.12.09.

láncszabály: ....

$f(g(x))\cdot g'(x)$ $\qquad$ $F(g(x))$

$$\int \underbrace{\cos(2x+5)}_{f(g(x))\cdot g'(x)}\,dx = \frac{1}{2}\cdot\int \cos(2x+5)\cdot 2\,dx = \frac{1}{2}\cdot\sin(2x+5)+C$$

$g(x) = 2x+5$ $\qquad$ $f(x) = \cos(x)$

$g'(x) = 2$ $\qquad$ $F(x) = \sin(x)$

$$\int \frac{1}{x-5}\cdot 1\,dx = \ln|x-5| + c$$

$f(g(x))\cdot 1$, $\quad g(x) = x-5$, $\quad 1 = g'$

$f(x) = \frac{1}{x} \rightarrow F(x)$

$$\int \frac{1}{(x-5)^6}\,dx = \frac{1}{(-5)\cdot(x-5)^5} + c$$

$f(g(x))$, $\quad g(x) = x-5$

$f(x) = \frac{1}{x^6} = x^{-6} \rightarrow F(x) = \int x^{-6}\,dx =$

$= \frac{x^{-5}}{-5} + c = \frac{1}{-5x^5} + c$

$$\frac{1}{6}\cdot\int \underbrace{\frac{1}{(x^6-5)}}_{f(g(x))} \cdot \underbrace{6x^5}_{g'} \, dx = \frac{1}{6} \ln|x^6-5| + c$$

$$f(g(x)) \rightarrow f(x) = \frac{1}{x} \longrightarrow F(x) = \ln|x|$$

$$g(x) = x^6-5 \longrightarrow g'(x) = 6\cdot x^5 - 0$$

$$\int \frac{\sin x}{\cos^6 x} \, dx = (-1)\int \underbrace{(\cos x)^{-6}}_{f(g(x))} \cdot \underbrace{(-\sin x)}_{g'(x)} \, dx = \cancel{(-1)} \cdot \underbrace{\frac{\cancel{-}1}{5\cdot(\cos x)^5}}_{F(g(x))} + c =$$

$$f = x^{-6} \longrightarrow F = \frac{1}{-5x^5} + c \qquad = \frac{1}{5\cdot\cos x^5} + c$$

$$g = \cos x \longrightarrow g'(x) = -\sin x$$

$$\int \underbrace{(\text{poli})}_{f} \cdot \underbrace{\begin{matrix} e^x \\ \sin x / \cos x \\ \operatorname{sh} x / \operatorname{ch} x \end{matrix}}_{g'} \, dx = f \cdot g - \int f' \cdot g$$

$$\int \underbrace{(x^2+x)}_{f} \cdot \underbrace{\operatorname{sh} x}_{g'} \, dx = (x^2+x) \cdot \operatorname{ch} x - \int \underbrace{(2x+1)}_{\color{red}{f}} \cdot \underbrace{\operatorname{ch} x}_{\color{red}{g'}} \, dx =$$

$$= (x^2+x) \cdot \operatorname{ch} x - \Big(\underbrace{(2x+1) \cdot \operatorname{sh} x - \int 2 \cdot \operatorname{sh} x \, dx}_{\color{red}{f \cdot g - \int f' \cdot g}} \Big) =$$

$$= (x^2+x) \operatorname{ch} x - (2x+1) \operatorname{sh} x + 2 \operatorname{ch} x + C$$

$$\int \underbrace{(5x+1)}_{f} \cdot \underbrace{e^{-2x}}_{g'} \, dx = \underbrace{(5x+1) \cdot \frac{e^{-2x}}{-2}}_{f \cdot g} - \int 5 \cdot \frac{e^{-2x}}{-2} \, dx = *$$

$\color{red}{(:2)}$

$$g' = e^{-2x} \rightarrow g = \color{red}{\frac{-1}{2}} \int e^{-2x} \, dx = -\frac{e^{-2x}}{2} + c$$

$$* = (5x+1)\cdot\frac{e^{-2x}}{-2} - \left(-\frac{5}{2}\right)\cdot\int e^{-2x}\,dx =$$

$$= (5x+1)\,\frac{e^{-2x}}{-2} + \frac{5}{2}\cdot\frac{e^{-2x}}{-2} + c$$

invers

$$\int \underbrace{x}_{g'}\cdot\underbrace{\ln(2x)}_{f}\,dx = \overset{f\,\cdot\,g}{\ln(2x)\cdot\frac{x^2}{2}} - \int \overset{g\,\cdot\,f'}{\frac{x^2}{\cancel{2}}\cdot\frac{1}{2x}\cdot\cancel{2}}\,dx$$

$$f' = (\ln 2x)' = \frac{1}{2x}\cdot 2$$

$$g = \int x\,dx = \frac{x^2}{2} + c$$

$$= \ln(2x)\cdot\frac{x^2}{2} - \int\frac{x}{2}\,dx = \ln(2x)\cdot\frac{x^2}{2} - \frac{x^2}{4} + c$$

$$d: \left(\ln 2x\cdot\frac{x^2}{2} - \frac{x^2}{4}\right)' = \frac{1}{\cancel{2}x}\cdot\cancel{2}\cdot\frac{x^2}{2} + \ln(2x)\cdot x - \frac{x}{2}$$

$$= \frac{x}{2}$$

$$\int \frac{1}{\operatorname{ch} x}\, dx = \int \frac{2}{e^x + e^{-x}}\, dx = \int \frac{2}{t + \frac{1}{t}} \cdot \underbrace{\frac{dx}{dt}}_{x'(t) = \frac{1}{t}} \cdot \boxed{dt} =$$

$$\longrightarrow t = e^x$$

$$\longrightarrow x = \ln t \qquad \frac{dx}{dt} = \frac{1}{t}$$

$$\int \frac{1}{\operatorname{ch} x} \cdot \operatorname{sh} x = \ln|\operatorname{ch} x| + c$$

$$f(g(x)) \cdot g' \qquad F(g(x))$$

$$= \int \frac{2}{t + \frac{1}{t}} \cdot \frac{1}{t}\, dt = \int \frac{2}{t^2 + 1}\, dt = 2 \cdot \operatorname{arctg} t + c =$$

$$\int \frac{1}{x^2 + 1}\, dx = \operatorname{arctg} x + c$$

$$= 2 \cdot \operatorname{arctg} e^x + c$$

$$\int_0^1 \frac{1}{\operatorname{ch} x}\, dx = \left[2 \operatorname{arctg} e^x\right]_0^1 = 2 \cdot \operatorname{arctg}(e) - 2 \cdot \underbrace{\operatorname{arctg} 1}_{= \frac{\pi}{4}}$$

$$\int_2^3 \frac{2x}{x^2-1}\,dx$$

① nem kell polinomosztás

② nevező szorzattalak

$x^2-1=0 \quad x=\pm 1$

$\quad\ \ ||$

$(x-1)(x+1)$

$$\frac{2x}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{Ax+A+Bx-B}{(x-1)(x+1)}$$

$2x = (A+B)x + A - B$

$A+B=2 \quad \rightarrow \quad A=B=1$

$A-B=0 \rightarrow A=B$

$$\int_2^3 \frac{1}{x-1} + \frac{1}{x+1}\,dx = \Big[\ln|x-1| + \ln|x+1|\Big]_2^3 =$$

$$= \ln(2) + \ln(4) - (\underbrace{\ln(1)}_{=0} + \ln(3)) = \ln 4 - \ln 3 + \ln 2$$

$$\int \frac{2x}{x^2+1}\,dx = \ln|x^2+1| + c$$

$$f(g(x)) \cdot g'$$

$$f = \frac{1}{x}$$

$$g = x^2+1 \qquad g' = 2x$$

$$\int \frac{2}{x^2+1}\,dx = 2 \int \frac{1}{x^2+1}\,dx = 2 \cdot \operatorname{arctg} x + c$$