Konzultáció P0 2024.12.16.

$$\int \underbrace{\cos^2(2x)}_{f(g(x))} \cdot \sin(2x)\, dx = \frac{1}{-2} \cdot \int \underbrace{\cos^2(2x)}_{f(g(x))} \cdot \underbrace{(-2\sin(2x))}_{g'}\, dx =$$

$$f(x) = x^2 \quad \longrightarrow \quad F(x) = \int x^2\, dx = \cancel{x}$$

$$g(x) = \cos(2x) \qquad g'(x) = -\sin(2x)\cdot 2$$

$$= -\frac{1}{2} \cdot \underbrace{(\cos(2x))^3/3}_{F(g(x)) \,=\, \frac{x^3}{3}} + c = \underline{-\frac{1}{6} \cdot \cos^3(2x) + c}$$

Ell:

$$\left(-\frac{1}{6} \cdot \cos^3(2x)\right)' = -\frac{1}{6} \cdot \underbrace{3\cos^2(2x)}_{f(g(2x))} \left(\cos(2x)\right)' =$$

$$= -\frac{1}{2} \cdot \cos^2(2x) \cdot -\sin(2x) \cdot (2x)' = \cos^2(2x) \cdot \sin(2x)$$

$$\int \underset{\text{invers}}{\overset{f \cdot g'}{\arcsin(3x)}}\,dx = \int \underbrace{1}_{g'} \cdot \underbrace{\arcsin(3x)}_{f}\,dx = *$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$\int f \cdot g'\,dx = f \cdot g - \int f' \cdot g\,dx$$

$$f = \arcsin(3x) \quad f' = \frac{1}{\sqrt{1-9x^2}} \cdot 3$$

$$g' = 1 \to g = x$$

$$* = x \cdot \arcsin(3x) - \int \frac{1}{\sqrt{1-9x^2}} \cdot \underbrace{3 \cdot x}\,dx =$$

$$\hat{g} \cdot \hat{f} \quad - \int f' \cdot g\,dx$$

$$= x \cdot \arcsin(3x) + \frac{1}{6} \int \underbrace{\frac{1}{\sqrt{1-9x^2}}}_{f(g(x))} \cdot \overbrace{(-18x)}^{g'(x)}\,dx = **$$

$$f = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}} \to F = \int x^{-\frac{1}{2}}\,dx = 2\sqrt{x} + c$$

$$g = 1 - 9x^2 \quad g' = (-18x)$$

$$** = \underbrace{x \cdot \arcsin(3x) + \frac{1}{6} \cdot 2\sqrt{1-9x^2} + c}_{F(g(x))}$$

$$\int \frac{1}{\cos^2(x)\cdot\sqrt{tg(x)}}\,dx = \int \underbrace{\frac{1}{\sqrt{tg(x)}}}_{f(g(x))} \cdot \underbrace{\frac{1}{\cos^2(x)}}_{g'(x)}\,dx =$$

$$f = \frac{1}{\sqrt{x}} \rightarrow F = 2\sqrt{x}$$

$$g = tg\,x \rightarrow g'(x) = \frac{1}{\cos^2 x}$$

$$= \underbrace{2\sqrt{tg(x)}}_{F(g(x))} + C$$

$$\left(2\sqrt{tg(x)}\right)' = \cancel{2}\cdot\frac{1}{\cancel{2}\sqrt{tg(x)}}\cdot\frac{1}{\cos^2 x} = \frac{1}{\cos^2 x\cdot\sqrt{tg\,x}}$$

$$(-1)\int \underbrace{\frac{1}{\cos^2 x}}_{f(g(x))} \cdot \underbrace{-\sin x}_{g'}\,dx = (-1)\cdot\frac{-1}{\cos x} + C = \frac{1}{\cos x} + C$$

$$F = \int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} + C = \frac{-1}{x} + C$$

$$\int \underset{f'}{e^{2x}} \cdot \underset{g}{\mathrm{sh}(x)}\, dx = \underbrace{\cdots\cdots\cdots}\; - c \cdot \int e^{2x} \cdot \mathrm{sh}\, x\, dx$$

$$(1+c) \int e^{2x} \cdot \mathrm{sh}\, x\, dx = \cdots\cdots$$

$$f \cdot g - \int f \cdot g'$$

$$\int \underbrace{(x^2+1)}_{g} \cdot \underbrace{e^{2x}}_{f'} dx = \frac{e^{2x}}{2} \cdot (x^2+1) - \int \frac{e^{2x}}{\cancel{2}} \cdot \cancel{2}x\, dx =$$

$$f' \rightarrow f = \frac{e^{2x}}{2}$$

$$g = x^2+1 \rightarrow g' = 2x$$

$$= \frac{1}{2} e^{2x} \cdot (x^2+1) - \int \underbrace{x}_{g} \cdot \underbrace{e^{2x}}_{f'} dx = \frac{1}{2} \cdot e^{2x} (x^2+1) -$$

$$f' \rightarrow f = \frac{e^{2x}}{2}$$

$$g = x \rightarrow g' = (x)' = 1$$

$$- \left(\frac{e^{2x}}{2} \cdot x - \int \frac{e^{2x}}{2} \cdot 1\, dx \right) = \frac{1}{2} \cdot e^{2x} \cdot (x^2+1) - \frac{e^{2x}}{2} \cdot x + \frac{1}{2} \int e^{2x} dx =$$

$$f \cdot g - \int f \cdot g'$$

$$= \frac{1}{2} \cdot e^{2x} \cdot (x^2+1) - \frac{1}{2} \cdot e^{2x} \cdot x + \frac{1}{4} \cdot e^{2x} + C$$

$$\int \underbrace{sh(x)}_{f'} \cdot \underbrace{(5x+5)}_{g}\, dx = \underbrace{ch\,x \cdot (5x+5)}_{f \cdot g} - \int ch\,x \cdot 5\,dx =$$

$f = ch(x)$ $\quad g' = 5$

$$= ch(x)(5x+5) - 5 \cdot sh\,x + C$$

Ell: $\left(ch(x) \cdot (5x+5) - 5 \cdot sh\,x\right)' =$

$$sh(x) \cdot (5x+5) + \cancel{ch \cdot 5} - 5 \cdot \cancel{ch}\,x$$

Rendőr - elv

$$\lim_{n\to\infty} b_n = ?$$

$$\boxed{a_n} \le b_n \le \boxed{c_n}$$

$$a_n \xrightarrow{n\to\infty} A \qquad c_n \xrightarrow{n\to\infty} A \qquad \text{ekkor } b_n \xrightarrow{n\to\infty} A$$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

$$\lim_{n\to\infty} \sqrt[n]{n^2+n+3} = 1$$

$$\sqrt[n]{3} : \{3, \sqrt{3}, \sqrt[3]{3} \dots \} \to 1$$

$$\sqrt[n]{n} \longrightarrow 1$$

$$\sqrt[n]{n^2} = (\sqrt[n]{n})^2 \longrightarrow 1^2 = 1$$

$$\underset{\downarrow \atop 1}{\sqrt[n]{n}} \le \underset{\downarrow \atop 1}{\sqrt[n]{n^2}} \le \sqrt[n]{n^2+n+3} \underset{n>3}{\le} \underbrace{\sqrt[n]{n^2+n^2+n^2}}_{\to 1} = \sqrt[n]{3n^2} = \underset{\downarrow \atop 1}{\sqrt[n]{3}} \cdot \underset{\downarrow \atop 1^2}{\sqrt[n]{n^2}}$$

$$\lim_{n\to\infty} \sqrt[n]{3^n - 2^n} = 3$$

$$\sqrt[n]{n} \to 1$$

$$\sqrt[n]{3^n} = 3 \to 3$$

$$\sqrt[n]{2^n} = 2 \to 2$$

$$\underbrace{\sqrt[n]{3^n - 3^n}}_{\sqrt[n]{0}} \le \underbrace{\sqrt[n]{3^n - \frac{3^n}{2}}}_{\sqrt[n]{\frac{3^n}{2}}} \le \sqrt[n]{3^n - 2^n} \le \sqrt[n]{3^n} = 3$$

$$\sqrt[n]{0} \downarrow 0$$

$$\sqrt[n]{\frac{3^n}{2}} = \frac{3}{\sqrt[n]{2}}, \quad \sqrt[n]{2} \to 1$$

$$\downarrow \frac{3}{1}$$

$$2^n \le \frac{3^n}{2} \le 3^n$$

$$2 \not\le \frac{3}{2}$$

$$4 \le \frac{9}{2} = 4,5$$

$$\vdots$$

$$.....$$

$$\left(\sqrt[n]{3^n + n^2} \right)$$