Konzultáció P0 2025.01.06.

Rendőr-elv

$$\lim_{n\to\infty} \sqrt[n]{2^n+3^n} \qquad \sqrt[n]{2^n+3^n} = b_n$$

$$\lim_{n\to\infty} \sqrt[n]{2^n} = \lim_{n\to\infty} 2 = 2$$

$$\sqrt{2^2+3^2} = \sqrt{13} \neq 2+3$$

$$a_n \le b_n \le c_n \quad \left(\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = H\right)$$

$$\to \lim_{n\to\infty} b_n = H$$

$$\underbrace{\sqrt[n]{2^n}}_{\downarrow\ 2} \le \underbrace{\sqrt[n]{3^n}}_{\downarrow\ H=3} \le \underbrace{\sqrt[n]{2^n+3^n}}_{\downarrow\ 3} \le \sqrt[n]{3^n+3^n} = \sqrt[n]{2\cdot 3^n} =$$

$$= \underbrace{\sqrt[n]{2}}_{\downarrow\ 1} \cdot 3 \qquad 1 \cdot 3 = H$$

$$\lim_{n\to\infty} \sqrt[n]{a^n} = a \qquad \lim_{n\to\infty} \sqrt[n]{2} = 1$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{1}{2}} = 1$$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

(nevezetes)

$$\lim_{n\to\infty} \sqrt[n]{n^2} = \lim_{n\to\infty} \left(\sqrt[n]{n}\right)^2 = 1^2 = 1$$

$n = 1$

$n \quad n^2-1$

$1 > 0$

$n = 2$

$n \quad n^2-1$

$2 < 3$

$n = 3$

$n \quad n^2-1$

$3 < 8$

$$\lim_{n\to\infty} \sqrt[n]{n^2-1} \qquad \left(0;\ \sqrt{3};\ \sqrt[3]{8};\ \sqrt[4]{15}\ \ldots \Rightarrow 1\right)$$

2. mo 1. mo

$$\underset{\downarrow\ 1}{\sqrt[n]{n}} \underset{(n>1)}{\le} \underbrace{\sqrt[n]{n^2-\frac{n^2}{2}}}_{\sqrt[n]{\frac{n^2}{2}} = \frac{\left(\sqrt[n]{n}\right)^2 \to 1}{\sqrt[n]{2} \to 1} \to 1} \underset{(n>1)}{\le} \sqrt[n]{n^2-n} \le \sqrt[n]{n^2-1} \le \sqrt[n]{n^2} = \left(\sqrt[n]{n}\right)^2$$

$$\downarrow$$

$$1^2 = 1$$

12.17. vizsga. 6. fel.

$$\int_1^2 \frac{x^3+2x+1}{x^2+x}\,dx$$

$$\int \frac{x^3+2x+1}{x^2+x}\,dx$$

1 polinomosztás

$$x^3+2x+1 : x^2+x = x-1$$
$$-\;x^3+x^2$$
$$0 -x^2+2x+1$$
$$-\;-x^2-x$$
$$3x+1$$

$$\int \frac{x^3+2x+1}{x^2+x}\,dx = \int \frac{(x-1)(x^2+x)+3x+1}{x^2+x}\,dx =$$

$$= \int x-1+\frac{3x+1}{x^2+x}\,dx = \frac{x^2}{2}-x+\int\frac{3x+1}{x^2+x}\,dx =$$

$$\int \frac{3x+1}{x\cdot(x+1)}\,dx = \int \frac{A}{x} + \frac{B}{x+1}\,dx = *$$

$$\frac{3x+1}{x\cdot(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{A\cdot(x+1)+Bx}{x\cdot(x+1)}$$

$$3x+1 = A\,(x+1)+Bx$$

$$3 = A+B \qquad B = 2$$

$$1 = A \rightarrow A = 1$$

$$* \int \frac{1}{x} + \frac{2}{x+1}\,dx = \ln|x| + 2\cdot\ln|x+1| + c$$

$$\int \frac{x^3+2x+1}{x^2+x}\,dx = \frac{x^2}{2} - x + \ln|x| + 2\ln|x+1| + C$$

$$\int_1^2 \frac{x^3+2x+1}{x^2+x}\,dx = \left[\frac{x^2}{2} - x + \ln|x| + 2\ln|x+1|\right]_1^2 =$$

$$= \left(\cancel{\frac{2^2}{2} - 2} + \ln 2 + 2\ln 3\right) - \left(\frac{1^2}{2} - 1 + \underbrace{\ln 1}_{=0} + 2\cdot\ln 2\right)$$

$$= \frac{1}{2} - \ln 2 + 2\cdot\ln 3 = \frac{1}{2} + \ln\frac{9}{2}$$

$$\int \frac{3x+1}{x^2+x}\,dx = \int \frac{1}{x^2+x}\cdot(2x+1)\,dx + \int \frac{x}{x^2+x}\,dx$$

$$= \ln|x^2+x| + \ln|x+1| + C$$

$D_f \longrightarrow$ 1p

$\left.\begin{array}{l}\text{zérus}\\ \text{spec. hel.}\end{array}\right\}$ 1p

határérték $\longrightarrow$ 2p (3p)

$f' \to$ szél. + mon. $\longrightarrow$ 2p

$f'' \to$ konv. + infl $\longrightarrow$ 2p

vázl $\longrightarrow$ 1p

$R_f \longrightarrow$ 1p

$$\int \frac{1}{\operatorname{ch} x}\,dx = \int \frac{1}{\frac{e^x+e^{-x}}{2}}\,dx = \int \frac{2}{e^x+\frac{1}{e^x}}\,dx = \qquad t=e^x$$

$$= \int \frac{2}{t+\frac{1}{t}}\,\boxed{dx} = \int \frac{2t}{t^2+1}\cdot\boxed{\frac{dx}{dt}}\,dt = \qquad ! = \frac{1}{t}$$

$$t=e^x \quad (t>0) \qquad x(t)=\,? \qquad x'(t)=\frac{dx}{dt}=(\ln t)'_t=\frac{1}{t}$$

$$\ln t = \ln e^x$$

$$\ln t = x$$

$$= \int \frac{2\cancel{t}}{t^2+1}\cdot\frac{1}{\cancel{t}}\,dt = \int \frac{2}{t^2+1}\,dt = 2\cdot \operatorname{arctg}(t)+c$$

$$\Big\downarrow\ t=e^x$$

$$2\operatorname{arctg}(e^x)+c$$