Konzultáció P0 2025.01.13.

$$\frac{1}{2}\cdot\int \underbrace{\sin^3(2x)}_{f(g(x))} \cdot 2\cos(2x)\, dx = \frac{1}{2}\cdot\frac{\sin^4(2x)}{4} + c$$

$f(x) = x^3$ $\quad \sim g'$

$$\int f(g(x))\cdot g'(x)\,dx = F(g(x)) + c$$

$$\int f = F + c$$

$g(x) = \sin(2x) \rightarrow g'(x) = \cos(2x)\cdot 2$

$$\int f\,dx = \int x^3\,dx = \frac{x^4}{4} + c \qquad F(x) = \frac{x^4}{4}$$

$$\left(\frac{\sin^4(2x)}{8}\right)' = \frac{1}{8}\cdot 4\cdot\sin^3(2x)\cdot(\sin 2x)' =$$

$$= \cancel{\frac{1}{2}}\cdot\sin^3(2x)\cdot\cos(2x)\cdot\cancel{2}$$

$$\int \frac{1}{\ln^2(x)}\cdot\frac{1}{x}\,dx, \quad \int \ln(\operatorname{sh}(x))\cdot\operatorname{ch}(x)\,dx$$

Rendőr-elv $\qquad \lim\limits_{n\to\infty} b_n = ?$

$$\underset{\substack{n\to\infty \\ \downarrow \\ H}}{a_n} \le b_n \le \underset{\substack{n\to\infty \\ \downarrow \\ H}}{c_n} \qquad \text{ekkor } \lim_{n\to\infty} b_n = H$$

$$\sqrt[n]{n} \to 1$$

$$\sqrt[n]{2} \to 1 \qquad \sqrt[n]{\frac{1}{2}} \to 1 \qquad \frac{1}{\underset{\sqrt[n]{2}\,\to\, 1}{\sqrt[n]{2}}} \to 1$$

$$b_n = \sqrt[n]{2^n + n + 2} \quad ? \left\{ \begin{array}{l} \sqrt[n]{2^n} = 2 \to 2 \\ \sqrt[n]{n} \to 1 \\ \sqrt[n]{2} \to 1 \end{array} \right.$$

$$\sqrt[n]{2} \le \sqrt[n]{n} \le \sqrt[n]{n+2} \le \sqrt[n]{2^n} \le \sqrt[n]{2^n + n + 2} \le \sqrt[n]{2^n + 2^n + 2^n} = \sqrt[n]{3 \cdot 2^n} =$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \|$$

$$1 \le 1 \le 1 \le 2$$

$2^n \ge n$

$2^n \ge 2$

$$= \underbrace{\sqrt[n]{3}}_{\to 1} \cdot \underbrace{\sqrt[n]{2^n}}_{= 2}$$

$$1 \cdot 2$$

$$\Rightarrow \lim_{n \to \infty} \sqrt[n]{2^n + n + 2} = 2$$

Vizsga $\sqrt[n]{2^n + 3^n}$

$$\sqrt[n]{n^2 - \frac{n^2}{2}} \le \sqrt[n]{n^2 - n} \le \sqrt[n]{n^2} = \left(\sqrt[n]{n}\right)^2$$

$$\downarrow$$

$$1^2 = 1$$

$$\sqrt[n]{\frac{n^2}{2}} = \frac{\left(\sqrt[n]{n}\right)^2}{\sqrt[n]{2}} \begin{matrix} \to 1^2 \\ \to 1 \end{matrix} = 1$$

$$\lim_{n \to \infty} -\left(\frac{3}{5}\right)^n \le \lim_{n \to \infty} \frac{2^n - 3^n}{5^n} \le \lim_{n \to \infty} \frac{2^n}{5^n} = 0$$

$$= 0$$

Határozott integrál + parc. integrálás
(20. előadás)

$$\int_0^1 \underbrace{x}_{f} \cdot \underbrace{e^{-2x}}_{g'}\,dx = \left[x\cdot\frac{e^{-2x}}{-2}\right]_0^1 - \int_0^1 1\cdot\frac{e^{-2x}}{-2}\,dx = *$$

$$= \left[f\cdot g\right]_0^1 - \int_0^1 f'\cdot g\,dx$$

$$f = x \qquad f' = (x)' = 1$$

$$g' = e^{-2x} \qquad g = \frac{1}{-2}\int e^{-2x}\cdot(-2)\,dx = \frac{e^{-2x}}{-2} + C$$

$$= \left[-x\cdot\frac{e^{-2x}}{2}\right]_0^1 + \frac{1}{2}\cdot\int_0^1 e^{-2x}\,dx =$$

$$= \left[-x\cdot\frac{e^{-2x}}{2} + \frac{1}{2}\cdot\frac{e^{-2x}}{-2}\right]_0^1 = \left[e^{-2x}\cdot\frac{-x}{2} - e^{-2x}\cdot\frac{1}{4}\right]_0^1 =$$

$$= \underbrace{\left(e^{-2}\cdot\frac{-1}{2} - e^{-2}\cdot\frac{1}{4}\right)}_{-\frac{3}{4}\cdot\frac{1}{e^2}} - \underbrace{\left(\underset{1}{e^0}\cdot\frac{-0}{2} - \underset{1}{e^0}\cdot\frac{1}{4}\right)}_{-\frac{1}{4}} = -\frac{3}{4}\cdot\frac{1}{e^2} + \frac{1}{4}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \quad \dots \quad \lim_{n\to\infty}\left(1+\frac{k}{n}\right)^n = e^k$$

általánosan

$$\lim_{n\to\infty}\left(1+\frac{k}{\bullet}\right)^{\bullet} = e^k, \qquad \bullet \to \infty$$

pl: $\lim_{n\to\infty}\left(1-\frac{1}{3n}\right)^n = e^{-\frac{1}{3}}$

$$\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^n = e^3$$

pl: $\lim_{n\to\infty}\left(1+\frac{3}{n^2}\right)^{n^2} = e^3$

$$\lim_{n\to\infty}\left(1+\frac{3}{2n}\right)^{2n} = e^3 \text{ stb.}$$

$$\lim_{n\to\infty}\left(\frac{n-3}{n+2}\right)^{2n+1} = \lim_{n\to\infty}\left(\frac{n+2-5}{n+2}\right)^{2n+1} =$$

$$\lim_{n\to\infty}\left(\underbrace{\left(1-\frac{5}{n+2}\right)^{n+2}}_{e^{-5}}\right)^{\underbrace{\frac{2n+1}{n+2}}_{\substack{n\to\infty \\ \downarrow \\ 2}}} = \left(e^{-5}\right)^2 = e^{-10} = \frac{1}{e^{10}}$$

2-es strat. $\lim_{n\to\infty}\left(1-\frac{5}{n+2}\right)^{2n+1} = \lim_{n\to\infty}\left(1-\frac{5}{n+2}\right)^{2\cdot(n+2)-3} =$

$$= \lim_{n\to\infty}\left(\underbrace{\left(1-\frac{5}{n+2}\right)^{n+2}}_{e^{-5}}\right)^2\cdot\underbrace{\left(1-\frac{5}{n+2}\right)^{-3}}_{(1-0)^{-3}} = \left(e^{-5}\right)^2\cdot 1^{-3} = \frac{1}{e^{10}}$$

3-mas strat.

$$\lim_{n\to\infty}\left(\frac{n-3}{n+2}\right)^{2n+1} = \lim_{n\to\infty}\left(\frac{\frac{n-3}{n}}{\frac{n+2}{n}}\right)^{2n+1} =$$

$$\lim_{n\to\infty}\left(\frac{\left(1-\frac{3}{n}\right)}{\left(1+\frac{2}{n}\right)}\right)^{2n+1} = \lim_{n\to\infty}\frac{\left(\left(1-\frac{3}{n}\right)^{n}\right)^2\cdot\left(1-\frac{3}{n}\right)}{\left(\left(1+\frac{2}{n}\right)^{n}\right)^2\cdot\left(1+\frac{2}{n}\right)} = \frac{\left(e^{-3}\right)^2\cdot 1}{\left(e^{2}\right)^2\cdot 1} =$$

$$= \frac{e^{-6}}{e^4} = e^{-6-4} = e^{-10} = \frac{1}{e^{10}}$$

Parc. törtekre bontás

$$\int \frac{x+1}{x^2-4}\,dx$$

1. polinomosztás → most nem kell
2. nevezőt szorzattá bontjuk

$x^2-4=0 \quad x=\pm 2$

$x^2-4=(x+2)^1(x-2)^1$

$$\frac{x+1}{x^2-4}=\frac{x+1}{(x+2)(x-2)}=\frac{A}{x+2}+\frac{B}{x-2} \qquad A,B\in\mathbb{R}$$

$$x+1=A\cdot(x-2)+B\cdot(x+2)$$

$$1\cdot x+1=A\cdot x-2A+Bx+2B$$

$1=A+B \rightarrow A=1-B$

$1=-2A+2B$

$1=-2(1-B)+2B$

$1=-2+2B+2B$

$3=4B \rightarrow B=3/4$

$A=1-\frac{3}{4}=\frac{1}{4}$

$$\frac{x+1}{x^2-4}=\frac{1/4}{x+2}+\frac{3/4}{x-2}$$

$$\int \frac{x+1}{x^2-4}\,dx = \frac{1}{4}\int \cdot \frac{1}{x+2}\,dx + \frac{3}{4}\cdot\int \frac{1}{x-2}\,dx =$$

$$\int \frac{1}{x} = \ln|x| + c$$

$$= \frac{1}{4}\cdot \ln|x+2| + \frac{3}{4}\cdot \ln|x-2| + c$$

$$\int \frac{x+1}{x^2+4}\,dx = \underbrace{\frac{1}{2}\int \frac{1}{x^2+4}\cdot 2x\,dx}_{\int \frac{x}{x^2+4}\,dx} + \int \frac{1}{x^2+4}\,dx =$$

szorzatra tovább nem bontható

$$\sim \int \frac{1}{x^2+1}\,dx = \operatorname{arctg}(x) + c$$

$$= \frac{1}{2}\cdot \ln|x^2+4| + \frac{1}{4}\,2\cdot\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\cdot \frac{1}{2}\,dx =$$

$$F = \operatorname{arctg} \leftarrow \quad f = \frac{1}{x^2+1} \quad g = \frac{x}{2} \quad g' = \frac{1}{2}$$

$$= \frac{1}{2}\cdot \ln|x^2+4| + \frac{1}{2}\cdot \operatorname{arctg}\left(\frac{x}{2}\right) + c$$