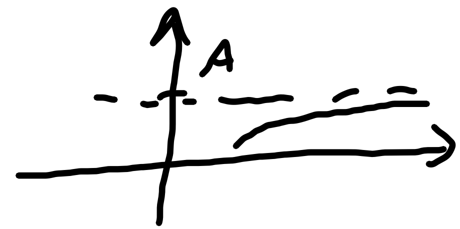
Konzultáció P0 2025.01.20.

Aszimptota $\pm\infty$-ben

- vízszintes $\lim\limits_{x\to\infty} f(x) = A \to \quad y = A$



ha $\lim\limits_{x\to\infty} f(x) = \pm\infty$

↳ esetleg lehet aszimptota!

$\lim\limits_{x\to\infty} \frac{f(x)}{x} = a \to$ meredekség : $y = ax + b$

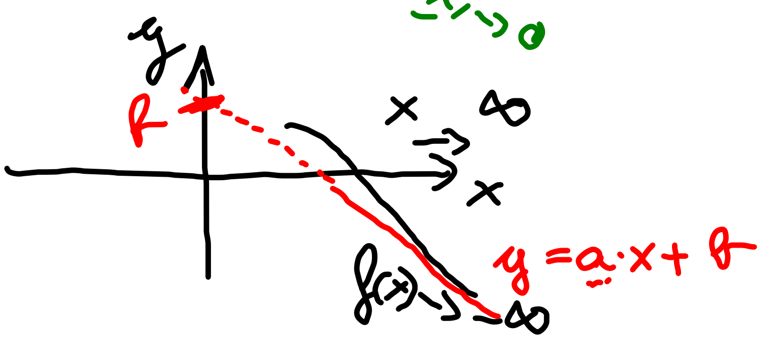
$b = \lim\limits_{x\to\infty} f(x) - ax$ kiszámolható

$$f(x) = \frac{x^2+1}{2-x} \to \frac{f(x)}{x} = \frac{x^2+1}{(2-x)x} = \frac{x^2+1}{2x-x^2}$$

$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\overset{\to\infty}{x^2+1}}{\underset{\to\infty}{2x}-\underset{\to\infty}{x^2}} = *$$

(vizsgálódunk, mert $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\overset{\to\infty+1}{x^2+1}}{\underset{\to 2-\infty}{2-x}} = "\frac{\infty}{-\infty}" =$

$= \lim_{x\to\infty} \frac{\overset{\to\infty}{x} + \overset{\to 0}{\frac{1}{x}}}{\underset{\to 0}{\frac{2}{x}} - 1} = \frac{\infty+0}{0-1} = -\infty$)



$$* = "\frac{\infty}{-\infty}" = \lim_{x\to\infty} \frac{1+\frac{1}{x^2}}{\frac{2}{x}-1} = \frac{1+0}{0-1} = -1$$

"a"

$$y = (-1)\cdot x + b$$

$$b = \lim_{x\to\infty} f(x) - \underbrace{a}_{-1}x = \lim_{x\to\infty} \frac{x^2+1}{2-x} - (-1)x =$$

$$= \lim_{x\to\infty \, (-\infty)} \frac{x^2+1}{2-x} + \frac{x}{1} = \lim_{x\to\infty} \frac{x^2+1+x(2-x)}{2-x} =$$

$$= \lim_{x\to\infty \, (-\infty)} \frac{\cancel{x^2}+1+2x-\cancel{x^2}}{2-x} = \lim_{x\to\infty \, (-\infty)} \frac{1+2x}{2-x} = \frac{\text{"}\infty}{-\infty}\text{"} = \left(\text{"}\frac{-\infty}{\infty}\text{"}\right)$$

$1+2x \to \infty+1$, $2-x \to 2-\infty$

$$= \lim_{x\to\infty \, (-\infty)} \frac{\frac{1}{x}+2}{\frac{2}{x}-1} = \frac{2}{-1} = -2$$

$\frac{1}{x}\to 0$, $\frac{2}{x}\to 0$

$(-\infty\text{-ben is})$

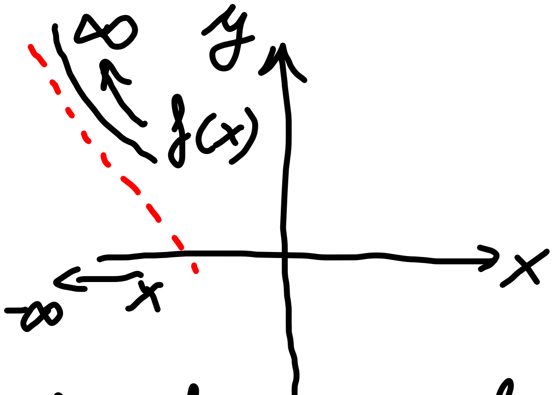
$\hookrightarrow \quad y = (-1)x - 2 =$

$$= -x-2$$

$f(x) = \dfrac{x^2+1}{2-x}$ $\quad -\infty$-ben ?

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{\overbrace{x^2+1}^{\to\infty}}{\underbrace{2-x}_{\to\infty}} = \text{"}\frac{\infty}{\infty}\text{"} =$$

$$= \lim_{x\to-\infty} \frac{\overbrace{x}^{\to-\infty} + \overbrace{\frac{1}{x}}^{\to 0-}}{\underbrace{\frac{2}{x}}_{\to 0-} - 1} = \frac{-\infty}{-1} = \infty$$



↳ ezért ferde asz. keresése

$$a = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{x^2+1}{(2-x)\cdot x} = \lim_{x\to-\infty} \frac{x^2+1}{2x-x^2} = \text{"}\frac{\infty}{-\infty}\text{"} =$$

$$= \lim_{x\to-\infty} \frac{1+\overbrace{\frac{1}{x^2}}^{\to 0+}}{\underbrace{\frac{2}{x}}_{\to 0-} - 1} = \frac{1}{-1} = -1 = a \qquad y = -x + b$$

$$b = \lim_{x\to-\infty} f(x) + x = \lim_{x\to-\infty} \frac{x^2+1}{2-x} + x = *$$

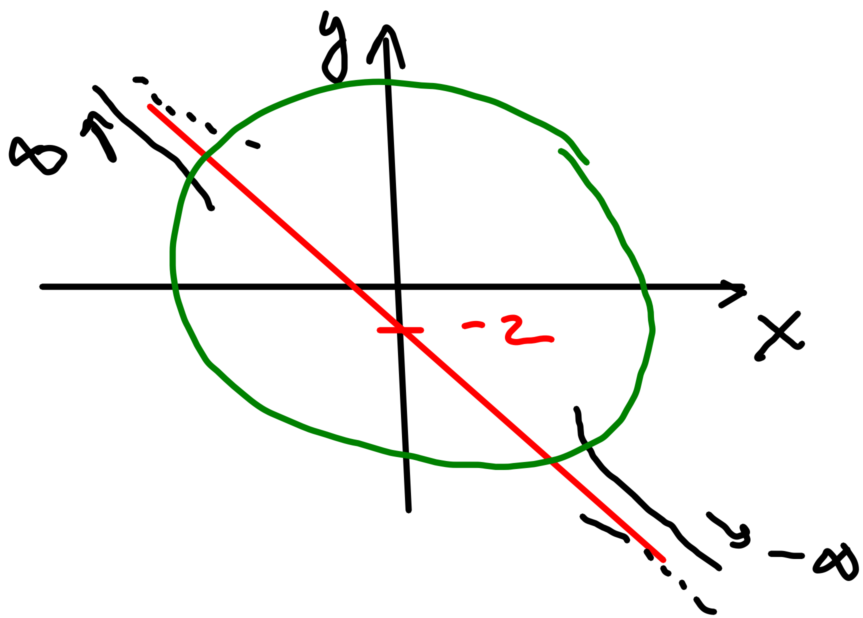
$$* = \lim_{x\to-\infty} \frac{x^2+1}{2-x} + x = \lim_{x\to-\infty} \frac{x^2+1+x(2-x)}{2-x} =$$

$$= \lim_{x\to-\infty} \frac{1+2x}{2-x} = \frac{-\infty}{\infty} = \lim_{x\to-\infty} \frac{\overset{\to 0^-}{\frac{1}{x}}+2}{\underset{\to 0^-}{\frac{2}{x}}-1} = \frac{2}{-1} = -2$$

ezért $y=-x-2$ egyenes

a $-\infty$-ben is ferde asz. $f(x)$-nek

$y=-x-2$



11 gy F2 / b

$$\int \frac{x+2}{2x^2+5}\,dx$$

Parc. törtekre bontás

1. nincs polinomosztás
2. nevező nem bontható szorzatra!

$2x^2+5=0$ $(D<0)$

$x^2 \neq -\frac{5}{2}$

$$\int \underbrace{\frac{x+2}{2x^2+5}}_{g(x)}\,dx = \underbrace{\frac{1}{4}\cdot\int\frac{4\cdot x}{2x^2+5}\,dx}_{\text{egysz. hely.}} + \underbrace{\int\frac{2}{2x^2+5}\,dx}_{?}$$

$g'(x)=(2x^2+5)'=4x$

$$\frac{1}{4}\int\frac{4x}{\underbrace{\boxed{2x^2+5}}}\,dx = \boxed{\frac{1}{4}\cdot\ln|2x^2+5|+c}$$

$F(g(x))$

$f(g(x)) \to \frac{1}{\boxed{g(x)}}\cdot g'(x)$

$f(x)=\frac{1}{x}$

$F(x)=\int\frac{1}{x}\,dx=\ln|x|+c$

$$\int \frac{2}{2x^2+5}\,dx = 2\int \frac{1}{2x^2+5} = \frac{2}{5}\cdot\int \frac{1}{\frac{2x^2}{5}+1}\,dx =$$

$$\int \frac{1}{\underset{g(x)}{x}^2+1}\,dx = \operatorname{arctg}(x)+c$$

$$g(x)^2 = \frac{2x^2}{5}$$

$$g(x) = \sqrt{\frac{2}{5}}\cdot x \qquad g'(x) = \sqrt{\frac{2}{5}}$$

$$\sqrt{\frac{5}{2}} \qquad = \frac{2}{5}\cdot\int \underbrace{\frac{1}{\left(\sqrt{\frac{2}{5}}x\right)^2+1}}_{f(g(x))}\cdot\underbrace{\sqrt{\frac{2}{5}}}_{g'(x)}\,dx = \sqrt{\frac{5}{2}}\cdot\frac{2}{5}\cdot\underbrace{\operatorname{arctg}\left(\sqrt{\frac{2}{5}}x\right)}_{F(g(x))}+c = *$$

$$f(x) = \frac{1}{x^2+1} \rightarrow F(x) = \int f(x)\,dx = \operatorname{arctg}(x)+c$$

$$* = \sqrt{\frac{2}{5}}\cdot\operatorname{arctg}\left(\sqrt{\frac{2}{5}}x\right)+c$$

$$\int \frac{x+2}{2x^2+5}\,dx = \underbrace{\frac{1}{4}\cdot \ln|2x^2+5| + \sqrt{\frac{2}{5}}\cdot \operatorname{arctg}\left(\sqrt{\frac{2}{5}}\,x\right)+c}_{\text{primitív fgv.}}$$

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jan. 7. vizsga

5. fel:

a, $$\int \underbrace{\frac{1}{\ln^2(x)}}_{f(g(x))} \cdot \underbrace{\frac{1}{x}}_{g'(x)}\,dx = -\frac{1}{\ln(x)} + c$$

$$F(g(x)) + c$$

$$g(x) = \ln x \quad g'(x) = \frac{1}{x}$$

$$f(x) = \frac{1}{x^2} \rightarrow F = \int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} + c = -\frac{1}{x} + c$$

b)

$$\int \underbrace{(x+1)}_{f(x)} \underbrace{\sin(2x)}_{g'(x)} dx = (x+1)\cdot\frac{-\cos 2x}{2} - \underbrace{\int 1\cdot\frac{-\cos 2x}{2} dx}_{} = *$$

$$f\cdot g - \int f'\cdot g$$

$$f(x) = x+1 \longrightarrow f' = (x+1)' = 1$$

$$g'(x) = \sin(2x) \longrightarrow g(x) = \int g'(x)\,dx =$$

$$= \frac{1}{2}\int \sin(2x)\cdot 2\,dx = \frac{1}{2}\cdot -\cos(2x) + C$$

$$(-\cos x)' = \sin x$$

$$(-\cos(2x))' = \sin(2x)\cdot 2$$

$$* = -\frac{(x+1)\cdot\cos 2x}{2} + \frac{1}{2}\frac{1}{2}\cdot\int \underbrace{\cos(2x)\cdot 2}_{}\,dx = -\frac{(x+1)\cos 2x}{2} + \frac{1}{4}\cdot\sin(2x) + C$$

6.) $\displaystyle\int_0^3 \frac{5x}{x^2-3x-4}\,dx$

① nem kell polinomosztás

② nevező szorzatalakja

$x^2-3x-4=(x-4)^1(x+1)^1$

$x_1=4 \quad x_2=-1$

$\rightarrow$ részlettörtek: $\dfrac{A}{x-4}+\dfrac{B}{x+1}=\dfrac{A(x+1)+B(x-4)}{(x-4)(x+1)}$

$$5x = A(x+1)+B(x-4)$$

$$5x+\underbrace{0}=(A+B)x+\underbrace{A-4B}$$

$\left.\begin{aligned}5&=A+B\\0&=A-4B\end{aligned}\right\}$ $\quad 5=4B+B=5B \Rightarrow B=1$

$A=4B \qquad A=4$

$$\int_0^3 \frac{5x}{x^2-3x-4}\,dx=\int_0^3\frac{4}{x-4}\,dx+\int_0^3\frac{1}{x+1}\,dx=*$$

$$\int_0^3 \frac{4}{x-4}\,dx = 4\int_0^3 \frac{1}{x-4}\cdot 1\,dx = 4\left[\ln|x-4|\right]_0^3 =$$

$g(x)$ $\quad g'(x)=1$

$f(x)=\frac{1}{x}$ $\quad F(x)=\ln|x|+c$

$$= 4\cdot\left(\underbrace{\ln|3-4|}_{\ln(1)=0} - \underbrace{\ln|0-4|}_{\ln(4)}\right) = -4\cdot\ln(4)$$

$$\int_0^3 \frac{1}{x+1}\cdot 1\,dx = \left[\ln|x+1|\right]_0^3 = \ln|3+1| - \underbrace{\ln(1)}_{=0} = \ln(4)$$

$$\int_0^3 \frac{5x}{x^2-3x-4}\,dx = -4\ln(4) + \ln(4) = -3\ln(4)$$