Matematika A1 vizsg GTK PSz&NG MEGOLDÁSOK 2023.12.13.

1.) 5+4p

a) $$\lim_{n\to\infty} \frac{\sqrt{n}-\sqrt{n+3}}{2-\sqrt{n}} = \text{"}\frac{\infty-\infty}{-\infty}\text{"} = \lim_{n\to\infty} \frac{n-(n+3)}{(2-\sqrt{n})(\sqrt{n}+\sqrt{n+3})} = \frac{-3}{-\infty\cdot\infty} = 0$$

b) $$\lim_{n\to\infty} \left(\frac{n+1}{n-2}\right)^{3n} = \lim_{n\to\infty} \left(\frac{\left(1+\frac{1}{n}\right)^n}{\left(1-\frac{2}{n}\right)^n}\right)^3 = \left(\frac{e}{e^{-2}}\right)^3 = e^9$$

2.) 5p

$x_0 = 1$

$$f(1) = (1^2+1)\cdot\ln(2\cdot1-1) = 2\cdot\ln(1) = 0$$

$$f'(x) = (2x+1)\ln(2x-1) + (x^2+x)\frac{1}{2x-1}\cdot 2$$

$$f'(1) = (2\cdot1+1)\ln(1) + (1^2+1)\frac{1}{2\cdot1-1}\cdot2 = 0 + \frac{2\cdot2}{1} = 4$$

érintő: $y = 4\cdot(x-1) + 0 = 4x-4$

3.) 5p

$$D_f = \mathbb{R} \setminus \{1\}$$

$$\lim_{x\to 1+} \frac{\overbrace{2x^2-x}^{\to 1}}{\underbrace{x-1}_{\to 0+}} = \frac{1}{0+} = \infty \qquad \lim_{x\to 1-} \frac{\overbrace{2x^2-x}^{\to 1}}{\underbrace{x-1}_{\to 0-}} = \frac{1}{0-} = -\infty$$

függ. asz.: $x=1$

$$\lim_{x\to\infty} \frac{2x^2-x}{x-1} = \lim_{x\to\infty} \frac{\overbrace{2x-1}^{\to\infty}}{1-\underbrace{\frac{1}{x}}_{\to 0}} = \infty \qquad \text{vízsz. asz. nincs}$$

$$a = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{2x-1}{x-1} = \lim_{x\to\infty} \frac{2-\frac{1}{x}}{1-\frac{1}{x}} = 2$$

$$b = \lim_{x\to\infty} f(x) - 2\cdot x = \lim_{x\to\infty} \frac{2x^2-x}{x-1} - 2x = \lim_{x\to\infty} \frac{2x^2-x-2x^2+2x}{x-1} =$$

$$= \lim_{x\to\infty} \frac{x}{x-1} = 1$$

ferde asz.: $y = 2x+1$ a ∞-ben

$-\infty$-ben hasonlóan ugyanez adódik

4.) 10p

$D_f = \mathbb{R}$, páros

zérushelyek: $a^2 - 4a + 3 = 0 = (a-3)(a-1)$

$x^2 = 1 \quad x_{1,2} = \pm 1 \quad x^2 = 3 \quad x_{3,4} = \pm\sqrt{3}$

$\lim\limits_{x \to \infty} x^4 - 4x^2 + 3 = \infty$ ($-\infty$-ben is, mert páros)

$f'(x) = 4x^3 - 8x \Rightarrow 4x \cdot (x^2 - 2) = 0$

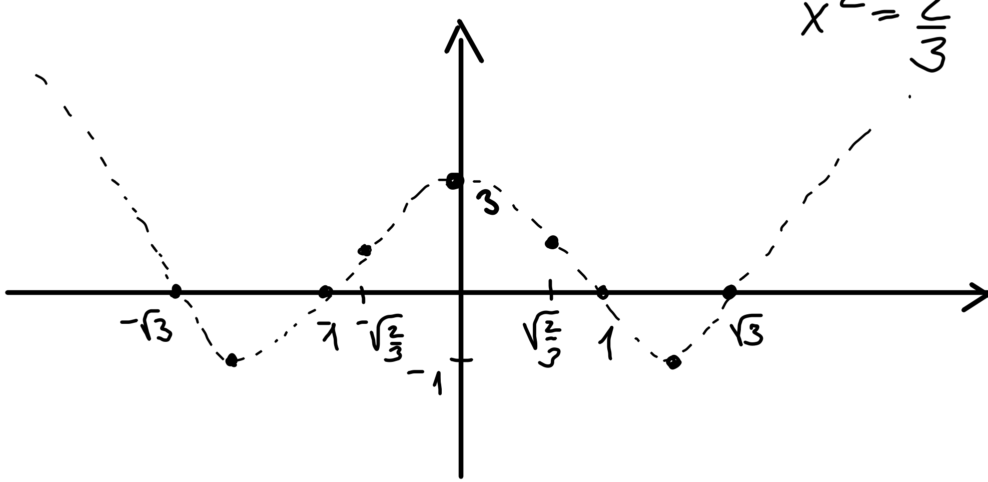
$x = 0 \qquad x = \pm\sqrt{2}$

x		$-\sqrt{2}$		0		$\sqrt{2}$	
f'	$-$	0	$+$	0	$-$	0	$+$
f	$\searrow$	MIN -1	$\nearrow$	MAX 3	$\searrow$	MIN -1	$\nearrow$

$f''(x) = 12x^2 - 8 \Rightarrow 4(3x^2 - 2) = 0$

$x^2 = \frac{2}{3} \Rightarrow x = \pm\sqrt{\frac{2}{3}}$

x		$-\sqrt{\frac{2}{3}}$		$\sqrt{\frac{2}{3}}$	
f''	$+$	0	$-$	0	$+$
f	$\cup$	I $\frac{7}{9}$	$\cap$	I $\frac{7}{9}$	$\cup$



R_f: $[-1, \infty)$

5.) 3+5p

a) $$\int \frac{1}{x\sqrt{\ln(x)}}\,dx = 2\sqrt{\ln(x)} + c$$

b) $$\int (x^2+1)\cos(2x)\,dx = (x^2+1)\frac{\sin(2x)}{2} - \int 2x\cdot\frac{\sin(2x)}{2}\,dx =$$

$$= (x^2+1)\frac{\sin(2x)}{2} + 2x\,\frac{\cos(2x)}{4} - \int 2\cdot\frac{\cos(2x)}{4}\,dx = (x^2+1)\frac{\sin(2x)}{2} + x\cdot\frac{\cos(2x)}{2} - \frac{\sin(2x)}{4} + c$$

6.) 8p

$$V = \pi\int_1^2 \left(2x-\frac{1}{x}\right)^2 dx = \pi\int_1^2 4x^2 - 4 + \frac{1}{x^2}\,dx = \pi\cdot\left[4\frac{x^3}{3} - 4x - \frac{1}{x}\right]_1^2 =$$

$$= \pi\cdot\left(\left(\frac{4\cdot 8}{3} - 8 - \frac{1}{2}\right) - \left(\frac{4}{3} - 4 - 1\right)\right) = \pi\cdot\left(\frac{13}{6} + \frac{11}{3}\right) = \frac{35}{6}\pi$$