Matematika A1 vizsg GTK PSz&NG MEGOLDÁSOK 2024.01.09.

1.) 4+4p

a) $$\lim_{x\to\infty}\frac{1+\sqrt{x}}{x^3-\sqrt{x}} = \text{"}\frac{\infty}{\infty-\infty}\text{"} = \lim_{x\to\infty}\frac{\overbrace{\frac{1}{x^3}}^{\to 0}+\overbrace{\frac{1}{\sqrt{x^5}}}^{\to 0}}{1-\underbrace{\frac{1}{\sqrt{x^5}}}_{\to 0}} = \frac{0}{1-0} = 0$$

b) $$\lim_{x\to 0}\frac{\sin(2x)}{\pi\cdot x} = \text{"}\frac{0}{0}\text{"} = \lim_{x\to 0}\underbrace{\frac{\sin(2x)}{2x}}_{\to 1}\cdot\frac{2}{\pi} = \frac{2}{\pi}$$

VAGY L'H

$$\lim_{x\to 0}\frac{\overbrace{\cos(2x)}^{\to 1}\cdot 2}{\pi} = \frac{2}{\pi}$$

2.) 5p

$$f(0) = (0^2+2\cdot 0-1)\cdot\cos(0) = (-1)\cdot 1 = -1$$

$$f'(x) = (2x+2)\cdot\cos(2x) + (x^2+2x-1)\cdot -\sin(2x)\cdot 2$$

$$f'(0) = (2\cdot 0+2)\cdot\cos(0) + (0^2+2\cdot 0-1)\cdot -\sin(0)\cdot 2 = 2\cdot 1 + (-1)\cdot 0\cdot 2 = 2$$

érintő: $y = 2\cdot(x-0) - 1 = 2x-1$

3.) 6p

$D_f: \mathbb{R} \setminus \{-1\}$ $\quad 2x+2=0$

$x=-1$

$$\lim_{x\to -1+} \frac{4x+1}{2x+2} = \frac{-3}{0+} = -\infty$$

$$\lim_{x\to -1-} \frac{4x+1}{2x+2} = \frac{-3}{0-} = \infty$$

függ. asz. $x=-1$-ben

$$\lim_{x\to \pm\infty} \frac{4x+1}{2x+2} = \text{"}\frac{\infty}{\infty}\text{"} = \lim_{x\to\pm\infty} \frac{4+\overset{\to 0}{\frac{1}{x}}}{2+\underset{\to 0}{\frac{2}{x}}} = \frac{4}{2} = 2 \Rightarrow$$

$+\infty$ és $-\infty$-ben vízszintes asz.: $y=2$

4.) 10p

$D_f: \mathbb{R}$ zérushely $x=0$ ($e^{-2x}>0$ mindig)

nem periodikus, nincs paritása

$$\lim_{x\to\infty} \frac{x}{e^{2x}} = \text{"}\frac{\infty}{\infty}\text{"} \underset{L'H}{\rightsquigarrow} \lim_{x\to\infty} \frac{1}{2\cdot e^{2x}} = \frac{1}{\infty} = 0$$

vízsz. asz. $y=0$

$$\lim_{x\to -\infty} \frac{x}{e^{2x}} = \text{"}\frac{-\infty}{0+}\text{"} = -\infty \qquad \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{1}{e^{2x}} = \infty$$

nincs ferde aszimptota

$$f'(x) = 1 \cdot e^{-2x} + x \cdot (-2) \cdot e^{-2x} = (1-2x)\underbrace{e^{-2x}}_{>0}$$

$$1-2x = 0$$
$$\frac{1}{2} = x$$

x	$(-\infty, \frac{1}{2})$	$1/2$	$(1/2, \infty)$
f'	$+$	0	$-$
f	$\nearrow$	MAX	$\searrow$

$$1-2x > 0$$
$$\frac{1}{2} > x$$

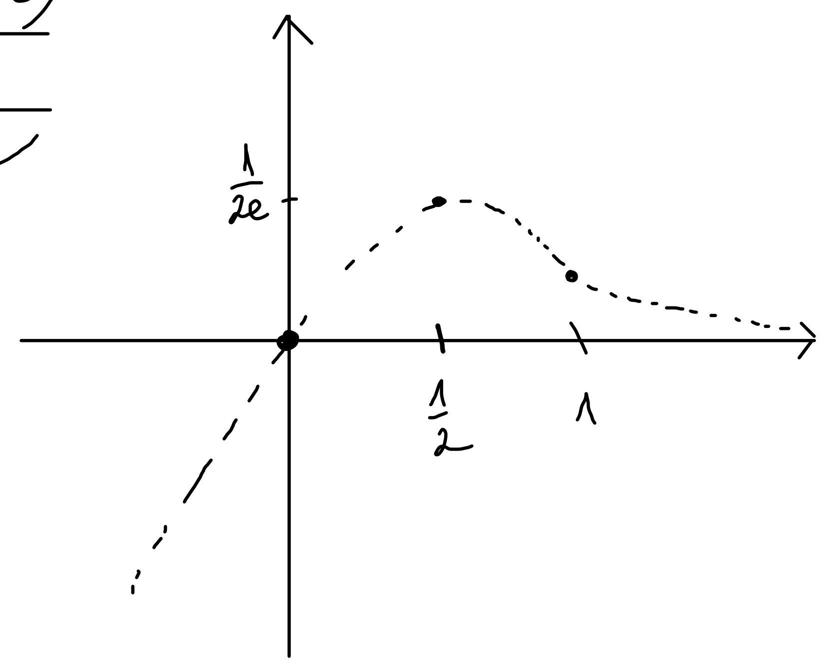
$$f\left(\frac{1}{2}\right) = \frac{1}{2e}$$

$$f''(x) = -2 \cdot e^{-2x} + (1-2x) \cdot (-2) \cdot e^{-2x} = e^{-2x} \cdot (-4 + 4x)$$

$$-4 + 4x = 0$$
$$x = 1$$

x	$(-\infty, 1)$	1	$(1, \infty)$
f''	$-$	0	$+$
f	$\frown$	Infl.	$\smile$

$$\mathcal{R}_f : \left(-\infty, \frac{1}{2e}\right]$$



5.) 4+5p

a, $\int \frac{x}{\sqrt{5x^2+6}}\,dx = \frac{1}{10}\cdot\int\frac{10x}{\sqrt{5x^2+6}}\,dx = \frac{1}{10}\cdot 2\sqrt{5x^2+6}+C$

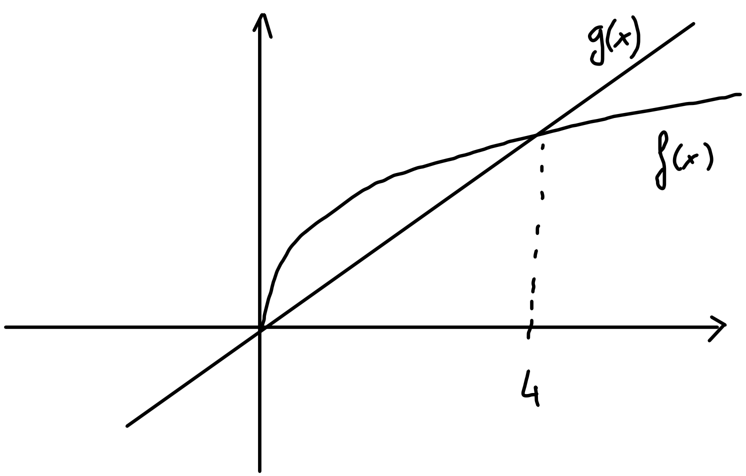
b, $\int \ln(2x)\cdot 1\,dx = \ln(2x)\cdot x - \int \frac{1}{2x}\cdot 2\cdot x\,dx =$

$= \ln(2x)\cdot x - x + C$

6.) 7p

$$f(x)=\sqrt{x}, \quad g(x)=\frac{x}{2}$$

$$\left.\begin{array}{l}\sqrt{x}=\dfrac{x}{2}\\[2ex] x=\dfrac{x^2}{4}\end{array}\right\} \quad \begin{array}{l}\nearrow x=0\\ \searrow x=4\end{array}$$



$$T=\int_0^4 \sqrt{x}-\frac{x}{2}\,dx = \left[\frac{\sqrt{x^3}}{3/2}-\frac{x^2}{4}\right]_0^4 =$$

$$=\left(\frac{2}{3}\cdot 8-4\right)-0=\frac{4}{3}$$