1.) 4+5p

a) $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{n-3} = \lim_{n\to\infty}\underbrace{\left(1-\frac{1}{n+1}\right)^{n+1}}_{\to e^{-1}}\cdot\underbrace{\left(\frac{n}{n+1}\right)^{-4}}_{\to 1^{-4}} = \frac{1}{e}$

b) $\lim_{n\to\infty}\sqrt[n]{n^2+2n} = 1$

$$\left(\sqrt[n]{n}\right)^2 \le \sqrt[n]{n^2+2n} \le \sqrt[n]{2\cdot n^2} = \sqrt[n]{2}\,\sqrt[n]{n}^{\,2}$$

$$\left(\sqrt[n]{n}\right)^2 \xrightarrow{n\to\infty} 1^2 = 1 \implies \sqrt[n]{n^2+2n} \xrightarrow{n\to\infty} 1 \impliedby \underbrace{\sqrt[n]{2}}_{\downarrow\, 1}\,\underbrace{\sqrt[n]{n}^{\,2}}_{\downarrow\, 1} \xrightarrow{n\to\infty} 1$$

2.) 5p

$D_f: \mathbb{R}\setminus\{-3\}$

injektív, mert $x_1 \neq x_2 \in D_f$-re

$\frac{2x_1-1}{x_1+3} = \frac{2x_2-1}{x_2+3}$ esetén

$(2x_1-1)(x_2+3) = (2x_2-1)(x_1+3)$

$2x_1x_2 - x_2 + 6x_1 - 3 = 2x_1x_2 - x_1 + 6x_2 - 3$

$6x_1 - x_2 = 6x_2 - x_1$

$7x_1 = 7x_2$ ellentmondás

$y = \frac{2x-1}{x+3}$

$yx + 3y = 2x - 1$

$3y + 1 = 2x - yx$

$3y + 1 = x(2-y) \quad y \neq 2$

$\frac{3y+1}{2-y} = x \Rightarrow f^{-1}(x) = \frac{3x+1}{2-x}$

$D_{f^{-1}}: \mathbb{R}\setminus\{2\}$

3.) 5p

$$f(2) = (2+1)\cdot e^{3\cdot 2-5} = 3\cdot e^{1} = 3e$$

$$f'(x) = 1\cdot e^{3x-5} + (x+1)\cdot e^{3x-5}\cdot 3$$

$$f'(2) = 1\cdot e^{3\cdot 2-5} + (2+1)\, e^{3\cdot 2-5}\cdot 3 = e + 3^2\cdot e = 10\cdot e$$

érintő:

$$y = 10e\cdot(x-2) + 3e = 10\cdot e\cdot x - 17\cdot e$$

4.) 10p

$D_f: \mathbb{R}\setminus\{-1\}$ zérushely: $x = \pm\sqrt{3}$ nincs spec. tul.

$$\lim_{x\to -1\pm} \frac{\overset{\to -2}{x^2-3}}{\underset{\searrow 0\pm}{x+1}} = \mp\infty$$ pólus, függ. asz.: $x=-1$

$$\lim_{x\to\pm\infty} \frac{x^2-3}{x+1} = \text{"}\frac{\infty}{\pm\infty}\text{"} = \lim_{x\to\pm\infty} \frac{x - \overset{\nearrow 0}{\frac{3}{x}}}{1+\underset{\searrow 0}{\frac{1}{x}}} = \pm\infty$$ vízsz. asz. nincs

$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x-\frac{3}{x}}{x+1} = 1 = a$$ ferde asz. $\pm\infty$-ben

$$b = \lim_{x\to\infty} \frac{x^2-3}{x+1} - x = \lim_{x\to\infty} \frac{\cancel{x^2}-3-\cancel{x^2}-x}{x+1} = -1$$ $y = x-1$

$$f'(x) = \frac{2x\cdot(x+1) - (x^2-3)\cdot 1}{(x+1)^2} = \frac{2x^2+2x-x^2+3}{(x+1)^2} =$$

$$= \frac{x^2+2x+3}{(x+1)^2} \Rightarrow f'(x) = 0 \quad \text{nincs}$$

$$x^2+2x+3=0$$

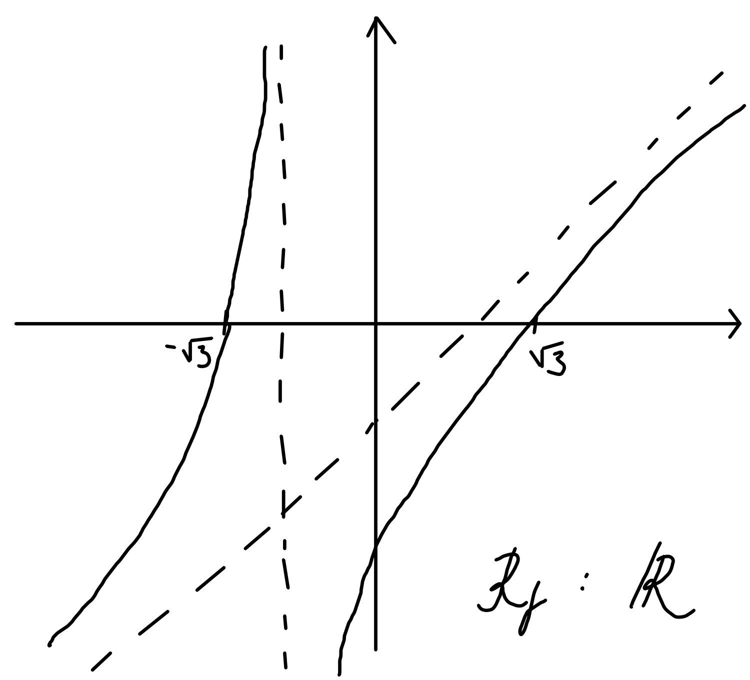
$$x_{1,2} = \frac{-2 \pm \sqrt{4-4\cdot 3}^{\,<0}}{2}$$

x	$(-\infty,-1)$	$(-1,\infty)$
f'	$+$	$+$
f	$\nearrow$	$\nearrow$

$$f''(x) = \frac{(2x+2)(x+1)^{\cancel{2}} - (x^2+2x+3)\cdot 2\cancel{(x+1)}}{(x+1)^{\cancel{4}\,3}} = \frac{2x^2+4x+2-2x^2-4x-6}{(x+1)^3} =$$

$$= \frac{-4}{(x+1)^3} \qquad f''(x) \neq 0$$

x	$(-\infty,-1)$	$(-1,\infty)$
f''	$\frac{-}{-} = +$	$\frac{-}{+} = -$
f	$\cup$	$\cap$



$R_f : \mathbb{R}$

5.) 8p $\displaystyle\int_1^2 \frac{x^3+2x}{x+1}\,dx$

$$
\begin{array}{l}
x^3+2x : (x+1) = x^2-x+3 \\
\ominus\, x^3+x^2 \\
\hline
-x^2+2x \\
\ominus\, -x^2-x \\
\hline
3x \\
\ominus\, 3x+3 \\
\hline
-3
\end{array}
$$

$$\int_1^2 \frac{x^3+2x}{x+1}\,dx = \int_1^2 x^2-x+3-\frac{3}{x+1}\,dx = \left[\frac{x^3}{3}-\frac{x^2}{2}+3x-3\ln|x+1|\right]_1^2 =$$

$$= \left(\frac{8}{3}-2+2\cdot 3-3\ln(3)\right)-\left(\frac{1}{3}-\frac{1}{2}+3-3\ln(2)\right)=\frac{7}{3}-\frac{3}{2}+3+3\ln\left(\frac{2}{3}\right) = \frac{23}{6}+3\ln\left(\frac{2}{3}\right)$$

6.) 8p $$V=\pi\cdot\int_0^{\pi/2}\cos^2(x)\,dx = \pi\cdot\int_0^{\pi/2}\frac{1+\cos(2x)}{2}\,dx =$$

$$= \pi\cdot\int_0^{\pi/2}\frac{1}{2}+\frac{\cos(2x)}{2}\,dx = \pi\left[\frac{1}{2}x+\frac{\sin(2x)}{4}\right]_0^{\pi/2} =$$

$$= \pi\left(\left(\frac{\pi}{4}+\frac{\overset{0}{\overset{\|}{\sin(\pi)}}}{4}\right)-\left(0+\frac{\overset{0}{\overset{\|}{\sin(0)}}}{4}\right)\right) = \pi\cdot\frac{\pi}{4} = \frac{\pi^2}{4}$$