1.) 7p $\lim_{n\to\infty} \frac{3n}{\sqrt{n+1}} = \text{"}\frac{\infty}{\infty}\text{"} = \lim_{n\to\infty} \frac{3\sqrt{n}}{\sqrt{1+\frac{1}{n}}} \to \frac{\infty}{1} = \infty$ $\Rightarrow$ felülről nem korlátos

$a_n \geq a_{n+1}$

$\frac{3n}{\sqrt{n+1}} \geq \frac{3n+3}{\sqrt{n+2}}$ a sorozat monoton növő

$n^2(n+2) \geq (n+1)^2(n+1)$

$n^3 + 2n^2 \geq n^3 + 3n^2 + 3n + 1$

$0 < n^2 + 3n + 1 \Rightarrow a_n < a_{n+1}$

} alulról korlátos, legnagyobb alsó korlát: $a_1 = \frac{3\cdot 1}{\sqrt{1+1}} = \frac{3}{\sqrt{2}}$

2.) 6p $D_f: \mathbb{R}\setminus\{0\}$ $x=0$ szakadás

$e^{2x} - 1 = 0$

$e^{2x} = 1$

$2x = 0$

$\lim_{x\to 0+} \frac{1}{\underbrace{e^{2x}-1}_{\to 1+}} = \frac{1}{0+} = \infty$

$\lim_{x\to 0-} \frac{1}{e^{2x}-1} = \frac{1}{0-} = -\infty$

} $\Rightarrow$ pólus

3.) 6p

$h(x) = 20 + 8\sqrt{x} - x \qquad x \geq 0$

$h'(x) = \frac{4}{\sqrt{x}} - 1 \qquad h'(x) = 0 \qquad \frac{4}{\sqrt{x}} = 1$

$\sqrt{x} = 4 \qquad x = 16 \quad (x \geq 0)$

x	$(0,16)$	16	$(16,\infty)$
h'	+	0	−
h	↗	MAX	↘

$h(16) = 20 + 8\sqrt{16} - 16 = 36$

36 ezer forint a max. haszon

4.) 10p

$D_f : \mathbb{R}$ zérushelyek : $-x \cdot (x^2+2x+1) = -x \cdot (x+1)^2$

$x_1 = 0 \; ; \; x_{2,3} = -1$

minden spec. tul.

$\lim_{x \to \infty} (-1)(x^3+2x^2+x) = -\infty \qquad \lim_{x \to -\infty} (-1)(x^3+2x^2+x) = \infty$

$$f'(x) = -3x^2 - 4x - 1$$

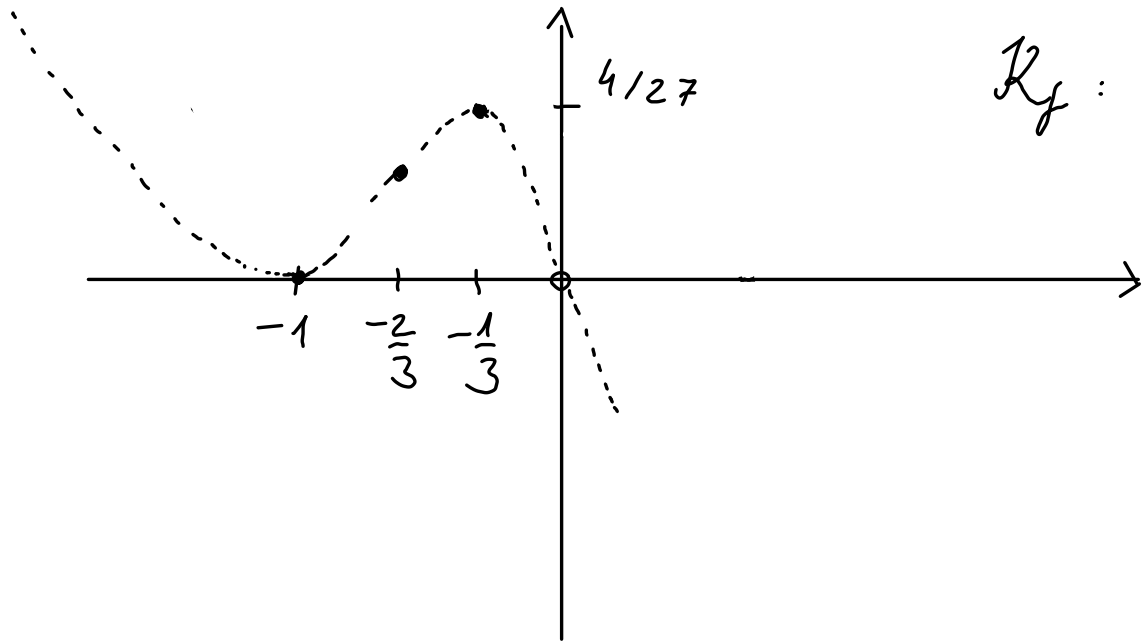
$$f'(x) = 0 \iff x_{1,2} = \frac{4 \pm \sqrt{16 - 4 \cdot 3}}{-6} \quad \nearrow -1 \quad \searrow -\frac{1}{3}$$

x	$(-\infty, -1)$	-1	$(-1, -\frac{1}{3})$	$-\frac{1}{3}$	$(-\frac{1}{3}, \infty)$
f'	$-$	0	$+$	0	$-$
f	$\searrow$	MIN	$\nearrow$	MAX	$\searrow$
		$f(-1) = 0$			$f(-\frac{1}{3}) = \frac{4}{27}$

$$f''(x) = -6x - 4$$

$$f''(x) = 0 \iff 6x = -4$$
$$x = -\frac{2}{3}$$

x	$(-\infty, -\frac{2}{3})$	$-\frac{2}{3}$	$(-\frac{2}{3}, \infty)$
f'	$+$	0	$-$
f	$\cup$	Infl.	$\cap$



$D_f : \mathbb{R}$

5.) 4+5p

a) $\int x^3 \cdot \sin(1+x^4)\,dx = -\frac{1}{4} \cdot \cos(1+x^4) + C$

b) $\int x^2 \cdot e^{-x} dx = -x^2 \cdot e^{-x} + \int 2x e^{-x} dx = -x^2 e^{-x} - 2x e^{-x} + \int 2e^{-x}\,dx =$

$= (-x^2 - 2x - 2)\, e^{-x} + C$

6.) 7p

$f'(x) = 2 \cdot \frac{3}{2} \cdot x^{\frac{1}{2}} = 3\sqrt{x}$

$(f'(x))^2 = 9x$

$J = \int_0^{1/3} \sqrt{1+9x}\,dx = \left[\frac{1}{9} \cdot \frac{2}{3} \cdot \sqrt{(1+9x)^3} \right]_0^{1/3} = \frac{2}{27} \cdot (8-1) = \frac{14}{27}$