Matematika A1 vizsg GTK PSz&NG MEGOLDÁSOK 2024.12.10.

1.) 5+3p

a) $$\lim_{n\to\infty} \frac{1}{\sqrt{2n+2}-\sqrt{2n+5}} = \text{"}\frac{1}{\infty-\infty}\text{"} = \lim_{n\to\infty} \frac{\sqrt{2n+2}+\sqrt{2n+5}}{2n+2-2n-5} = \lim_{n\to\infty} \frac{\sqrt{2n+2}+\sqrt{2n+5}}{-3} = \frac{\infty}{-3} = -\infty$$

b) $$\lim_{n\to\infty} \sqrt[n]{n^2+n+3} \quad \xrightarrow[\text{-elv}]{\text{Rendőr-}} \quad \underset{1^2}{\underset{\downarrow}{\sqrt[n]{n^2}}} \le \underset{1}{\underset{\downarrow}{\sqrt[n]{n^2+n+3}}} \underset{n>1}{\le} \sqrt[n]{3n^2} = \underbrace{\underset{1}{\underset{\downarrow}{\sqrt[n]{3}}} \cdot \underset{1^2}{\underset{\downarrow}{\left(\sqrt[n]{n}\right)^2}}}_{=1} \qquad (n\to\infty)$$

$$1^2 = 1 = 1$$

2.) 5p

$$x_0 = 0$$

$$f(0) = 3\cdot 0\cdot \sin(0) + (1-0)^3 = 0 + 1^3 = 1$$

$$f'(x) = 3\cdot \sin(2x) + 3x\cdot \cos(2x)\cdot 2 + 3\cdot(1-x)^2\cdot(-1) = 3\sin(2x) + 6x\cos(2x) - 3(1-x)^2$$

$$f'(0) = 3\cdot \underset{0}{\underset{\shortparallel}{\sin(0)}} + 6\cdot 0\cdot \underset{1}{\underset{\shortparallel}{\cos(0)}} - 3(1-0)^2 = -3\cdot 1 = -3$$

érintő: $y = -3(x-0) + 1 = -3x + 1$

3.) 6p

$k(x) = x + x + \frac{200}{x} + 5 = 2x + \frac{200}{x} + 5$ a kész. idő az előké-szítési idő x fgvében

$x > 0$

$k'(x) = 2 - \frac{200}{x^2} = \frac{2x^2 - 200}{x^2}$

$k'(x) = 0 \iff 2x^2 - 200 = 0 \Rightarrow x = \pm 10$, de $x > 0$

x	$(0,10)$	10	$(10,\infty)$
k'	$-$	0	$+$
k	$\searrow$	lok. min.	$\nearrow$

vagy $k''(x) = \frac{400}{x^3}$

$k''(10) = \frac{4}{10} > 0$

$\left.\begin{matrix} \\ \end{matrix}\right\}$ $x = 10$ perc előkészítés esetén $\Rightarrow$

$k(10) = 45$ perc a min. kész. idő.

4.) 10p

$D_f = \mathbb{R} \setminus \{1\}$; nincs spec. tul.

zérushelyek: $x^2 + 3x = x \cdot (x+3) \Rightarrow x_1 = 0$ és $x_2 = -3$

$\lim\limits_{x \to 1+} \frac{x^2+3x}{x-1} = \frac{4}{0+} = \infty$ $\lim\limits_{x \to 1-} \frac{x^2+3x}{x-1} = \frac{4}{0-} = -\infty$

$x = 1$ pólus $\to$ függ. asz.

$$\lim_{x\to\infty} \frac{x^2+3x}{x-1} = \infty \Rightarrow \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^2+3x}{x^2-x} = 1$$

$$\lim_{x\to\infty} f(x)-x = \lim_{x\to\infty} \frac{x^2+3x-x^2+x}{x-1} = 4$$

$$\lim_{x\to-\infty} \frac{x^2+3x}{x-1} = -\infty \Rightarrow \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{x^2+3x}{x^2-x} = 1$$

$$\lim_{x\to-\infty} f(x)-x = \lim_{x\to-\infty} \frac{x^2+3x-x^2+x}{x-1} = 4$$

$\pm\infty$-ben ferde asz.

$y = x+4$

$$f'(x) = \frac{(2x+3)(x-1)-(x^2+3x)}{(x-1)^2} = \frac{x^2-2x-3}{(x-1)^2}$$

$$x^2-2x+3 = 0 \Rightarrow x_1 = -1 \; ; x_2 = 3$$

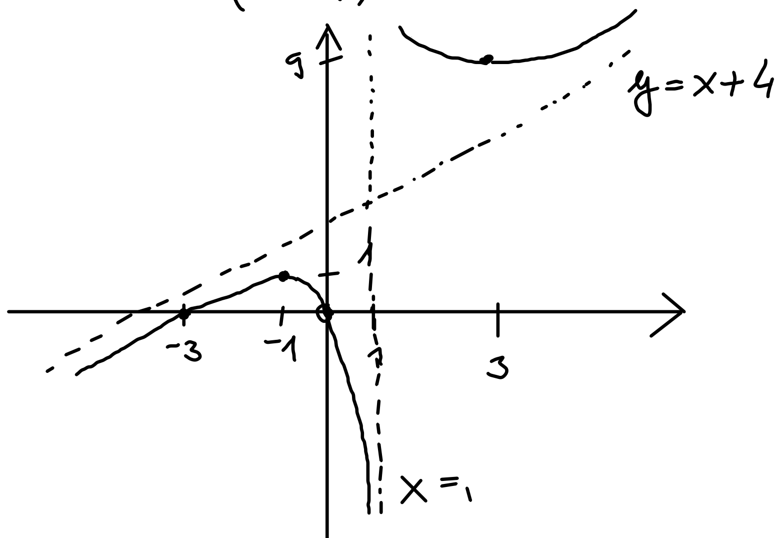
x	$(-\infty,-1)$	-1	$(-1,1)$	$(1,3)$	3	$(3,\infty)$
f'	+	0	−	−	0	+
f	↗	MAX	↘	↘	MIN	↗

$f(-1)=1$ $\quad$ $f(3)=9$

$$f''(x) = \frac{(2x-2)(x-1)^{\cancel{2}} - (x^2-2x-3)\cdot 2\cancel{(x-1)}}{(x-1)^{\cancel{4}3}} = \frac{8}{(x-1)^3} \to$$

x	$(-\infty,1)$	$(1,\infty)$
f''	−	+
f	∩	∪

nincs inflexiós pont



$R_f: (-\infty,1] \cup [9,\infty)$

5.) 5+3p

a) $$\int \frac{x+1}{x^2+3x-4}\,dx = \frac{3}{5}\cdot\int\frac{1}{x+4}\,dx + \frac{2}{5}\int\frac{1}{x-1}\,dx = \frac{3}{5}\cdot\ln|x+4| + \frac{2}{5}\cdot\ln|x-1| + c$$

$$x^2+3x-4=(x-1)(x+4) \qquad x+1 = A\cdot(x-1)+B(x+4) \qquad \left.\begin{array}{l} A+B=1 \\ 4B-A=1 \end{array}\right\} \begin{array}{l} B=\frac{2}{5} \\ A=\frac{3}{5} \end{array}$$

b) $$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx = 2\cdot\int \cos(\sqrt{x})\,\frac{1}{2\sqrt{x}}\,dx = 2\sin(\sqrt{x})+c$$

$$f(x)=\cos(x) \longrightarrow F(x)=\sin(x)$$
$$g(x)=\sqrt{x} \longrightarrow g'(x)=\frac{1}{2\sqrt{x}}$$

6.) 8p

$$\int_0^{\pi} (x^2+1)\cdot\sin(x)\,dx = \Big[-(x^2+1)\cdot\cos x + 2x\cdot\sin x + 2\cos x\Big]_0^{\pi} = *$$

$$\int (x^2+1)\sin x\,dx = (x^2+1)(-\cos(x)) + \int 2x\cdot\cos x\,dx = -(x^2+1)\cos x + 2x\cdot\sin x - \int 2\sin x\,dx =$$

$$= -(x^2+1)\cdot\cos x + 2x\cdot\sin x + 2\cos x + c$$

$$* = \big(-(\pi^2+1)\cdot(-1) + 0 + 2\cdot(-1)\big) - \big((-1)\cdot 1 + 0 + 2\big) = \pi^2 - 2$$