1.) 4+4p

a) $\lim\limits_{n\to\infty} \sqrt[n]{2^n+3^n+4^n}$ -> Rendőr-elv: $\sqrt[n]{4^n} \le \sqrt[n]{2^n+3^n+4^n} \le \sqrt[n]{3\cdot 4^n} = \sqrt[n]{3}\cdot 4$

$4 \le \lim = 4 \le 1\cdot 4$

b) $$\lim_{n\to\infty}\left(\frac{n-2}{n+1}\right)^{4n+1} = \lim_{n\to\infty}\left(\underbrace{\left(1-\frac{3}{n+1}\right)^{n+1}}_{\to e^{-3}}\right)^4 \cdot \underbrace{\left(1-\frac{3}{n+1}\right)^{-3}}_{\to 1} = \left(e^{-3}\right)^4\cdot 1 = e^{-12} = \frac{1}{e^{12}}$$

2.) 5p

$f(x) = e^{\frac{2}{1-x}}$ $D_f = \mathbb{R}\setminus\{1\}$ $x=1$-ben szakadás

$$\left.\begin{aligned}\lim_{x\to 1-} e^{\frac{2}{1-x}} &= e^{\frac{2}{0+}} = e^{\infty} = \infty\\ \lim_{x\to 1+} e^{\frac{2}{1-x}} &= e^{\frac{2}{0-}} = e^{-\infty} = 0\end{aligned}\right\}$$ másodfajú szakadás (nem pólus)

3.) 6p

$$k(x) = x + 20 - 4\sqrt{x} \qquad (x>0)$$

$$k'(x) = 1 - \frac{4}{2\sqrt{x}} = 1 - \frac{2}{\sqrt{x}} = \frac{\sqrt{x}-2}{\sqrt{x}}$$

$$\left(\begin{array}{l} k''(x) = \frac{1}{\sqrt{x^3}} \\ k''(4) = \frac{1}{8} > 0 \text{ MIN} \end{array} \right)$$

$$k'(x) = 0 \implies \sqrt{x} - 2 = 0$$

$$x = 4$$

	$(0,4)$	4	$(4,\infty)$
k'	$-$	0	$+$
k	$\searrow$	MIN	$\nearrow$

$x=4$ ezer forintért vegyen fát, akkor $24-8=16$ ezer forintot költ.

4.) 10p

$$f(x) = \frac{x^2+2x}{(x-1)^2}$$

$D_f = \mathbb{R} \setminus \{1\}$, zérushely: $x=0,-2$

nincs spec. tul.

$$\lim_{x \to 1\pm} \frac{x^2+2x}{(x-1)^2} = \frac{3}{0+} = \infty \to \text{pólus} \to \text{függ. asz}$$

$$x = 1$$

$$\lim_{x \to \infty} \frac{x^2+2x}{(x-1)^2} = \lim_{x \to \infty} \frac{1+\frac{2}{x}}{\left(1-\frac{1}{x}\right)^2} = 1 \to \text{vízszintes asz.}$$

$$y = 1$$

($-\infty$ ugyanígy)

$$f'(x) = \frac{(2x+2)(x-1)^{\cancel{2}} - (x^2+2x)\,2\cdot\cancel{(x-1)}}{(x-1)^{\cancel{4}\,3}} = \frac{-4x-2}{(x-1)^3}$$

$$f'(x) = 0 \iff -4x-2=0$$
$$x = -1/2$$

$$f\left(-\frac{1}{2}\right) = -\frac{1}{3}$$

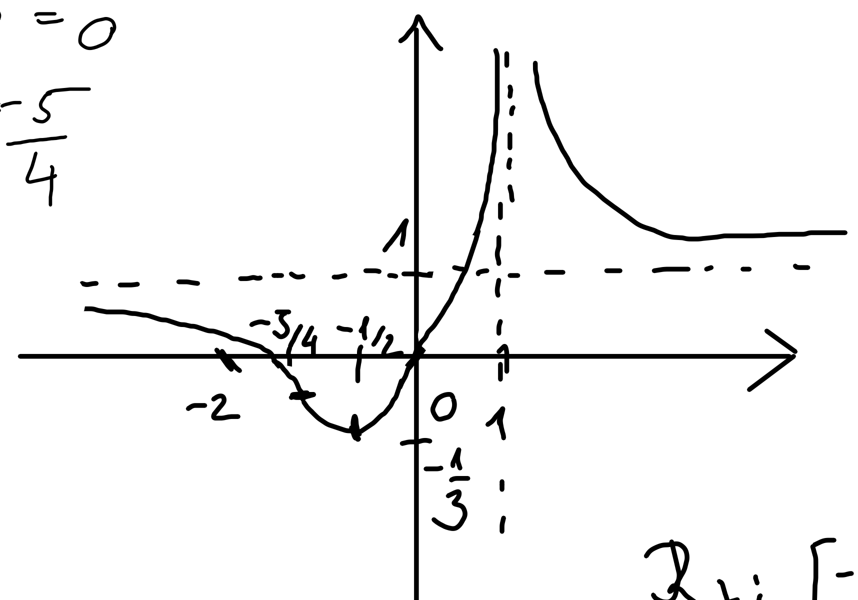
	$(-\infty,-\frac{1}{2})$	$-\frac{1}{2}$	$(-\frac{1}{2},1)$	$(1,\infty$
f'	$-$	0	$+$	$-$
f	↘	MIN	↗	↘

$$f''(x) = \frac{-4\cdot(x-1)^{\cancel{3}} - (-4x-2)\cdot 3\cancel{(x-1)}^{2}}{(x-1)^{\cancel{6}\,4}} = \frac{-4x+4+12x+6}{(x-1)^4} = \frac{8x+10}{(x-1)^4}$$

$$f''(x) = 0 \iff 8x+10=0$$
$$x = -\frac{5}{4}$$

	$(-\infty,-\frac{5}{4})$	$-5/4$	$(-\frac{5}{4},1)$	$(1,\infty)$
f''	$-$	0	$+$	$+$
f	∩	inf	∪	∪

$$f\left(-\frac{5}{4}\right) = -\frac{5}{27}$$



$$\mathcal{R}_f: \left[-\frac{1}{3}, \infty\right)$$

5.) 4+4p

a)
$$\int \frac{x^2+x+1}{x\sqrt{x}}\,dx = \int \sqrt{x} + \frac{1}{\sqrt{x}} + \frac{1}{x\sqrt{x}}\,dx = \frac{2}{3}\sqrt{x^3} + 2\sqrt{x} - \frac{2}{\sqrt{x}} + C$$

b)
$$\int \sin^3(2x)\cdot\cos(2x)\,dx = \frac{\sin^4(2x)}{2\cdot 4} + C = \frac{\sin^4(2x)}{8} + C$$

$$f = x^3 \rightarrow F = \frac{x^4}{4}$$

$$g = \sin(2x) \rightarrow g' = \cos(2x)\cdot 2$$

6.) 8p

$$\int_1^2 \frac{x^3+2x+1}{x^2+x}\,dx = \int_1^2 x-1+\frac{3x+1}{\underbrace{x^2+x}_{=x\cdot(x+1)}}\,dx = \left[\frac{x^2}{2}-x\right]_1^2 + \int_1^2 \frac{1}{x}+\frac{2}{x+1}\,dx =$$

$$x^3+2x+1 : x^2+x = x-1$$
$$x^3+x^2$$
$$-x^2+2x+1$$
$$-x^2-x$$
$$3x+1$$

$$\frac{A}{x}+\frac{B}{x+1} = \frac{3x+1}{x^2+x}$$

$$A+B=3 \rightarrow B=2$$
$$A=1$$

$$= \left[\frac{x^2}{2} - x + \ln|x| + 2\ln|x+1|\right]_1^2 = 0 + \ln 2 + 2\ln 3 + \frac{1}{2} - 2\ln 2 = \frac{1}{2} - \ln 2 + 2\ln 3$$