1.) 3+4p

a) $$\lim_{n\to\infty}\frac{\sqrt{n}+n^2}{1-n}=\lim_{n\to\infty}\frac{\underbrace{\frac{1}{\sqrt{n}}}_{\to 0}+\underbrace{n}_{\to\infty}}{\underbrace{\frac{1}{n}}_{\to 0}-1}=\frac{0+\infty}{0-1}=-\infty$$

b) $$\lim_{n\to\infty}\frac{2^n-3^n}{5^n}=\lim_{n\to\infty}\underbrace{\left(\frac{2}{5}\right)^n}_{\to 0}-\underbrace{\left(\frac{3}{5}\right)^n}_{\to 0}=0-0=0$$

2.) 7p

$$p(x)=-2x^4-x^3+5x^2-2x=x(-2x^3-x^2+5x-2)\longrightarrow x_1=0$$

$\hookrightarrow$ -2 osztói: ± 1; ± 2

$$p(1)=1\cdot(-2\cdot 1^3-1^2+5-2)=0\longrightarrow x_2=1$$

$$-2x^3-x^2+5x-2:(x-1)=-2x^2-3x+2$$
$$\begin{array}{l}
-2x^3+2x^2 \\
\hline
\quad -3x^2+5x-2 \\
\quad -3x^2+3x \\
\hline
\qquad\quad 2x-2 \\
\qquad\quad 2x-2 \\
\hline
\qquad\qquad 0
\end{array}$$

$$-2x^2-3x+2=0$$

$$x_{3,4}=\frac{3\pm\sqrt{9+16}}{-4}=\begin{cases}-2\\ \frac{1}{2}\end{cases}$$

$$p(x)=(-2)(x)(x-1)(x+2)\left(x-\frac{1}{2}\right)$$

3.) 5p

$$f(1) = 1^3 + \sqrt{1^2+3} = 3$$

érintő:

$$f'(x) = 3\cdot x^2 + \frac{1}{2\sqrt{x^2+3}}\cdot 2x$$

$$y = \frac{7}{2}(x-1)+3$$

$$f'(1) = 3\cdot 1^2 + \frac{2}{2\sqrt{1^2+3}} = \frac{7}{2}$$

$$y = \frac{7}{2}x - \frac{1}{2}$$

4.) 10p

$D_f: \mathbb{R} \setminus \{0\}$, zérushely nincs, nincs spec. tul.

$$\left.\begin{array}{l} \lim\limits_{x\to 0+} \dfrac{e^x}{x} = \dfrac{1}{0+} = \infty \\ \lim\limits_{x\to 0-} \dfrac{e^x}{x} = \dfrac{1}{0-} = -\infty \end{array}\right\}$$

pólus

$x=0$ (y-tengely)

függ. asz.

$$\lim_{x\to\infty} \frac{e^x}{x} = \text{"}\frac{\infty}{\infty}\text{"} \overset{L'H}{\rightsquigarrow} \lim_{x\to\infty} \frac{e^x}{1} = \infty$$

és $\lim\limits_{x\to\infty} \dfrac{e^x}{x^2} = \infty$ is

$\Rightarrow$ ∞-ben nincs asz.

$$\lim_{x\to -\infty} \frac{e^x}{x} = \frac{0+}{-\infty} = 0$$

$\Rightarrow$ $y=0$ (x-tengely) vízsz. asz.

a $-\infty$-ben

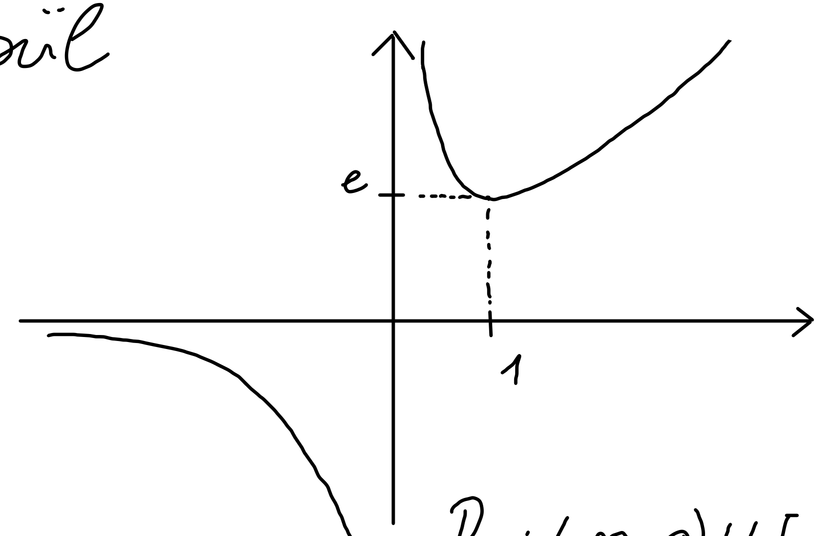
$$f'(x) = \frac{e^x \cdot x - e^x}{\underbrace{x^2}_{(>0)}} = \frac{e^x(x-1)}{x^2} \qquad f'(x)=0 \Leftrightarrow \underbrace{e^x}_{\neq 0}(x-1)=0 \quad x=1$$

	$(-\infty, 0)$	$(0, 1)$	1	$(1, \infty)$
f'	$-$	$-$	0	$+$
f	$\searrow$	$\searrow$	MIN $f(1) = e$	$\nearrow$

$$f''(x) = \frac{\left(e^x(x-1)+e^x\right)\cdot x^2 - e^x(x-1)\cdot 2x}{x^4} = \frac{\overbrace{e^x}^{>0} \cdot \overbrace{\left(x^2-2x+2\right)}^{>0 \text{ mindig mert } D<0}}{x^3}$$

$f''(x) = 0$ sosem teljesül

	$(-\infty, 0)$	$(0, \infty)$
f''	$-$	$+$
f	$\frown$	$\smile$



$R_f: (-\infty, 0) \cup [e, \infty)$

5.) 3+5p

a)
$$\int \frac{1}{\ln^2(x)} \cdot \frac{1}{x}\, dx = -(\ln(x))^{-1} + c = \frac{-1}{\ln(x)} + c$$

$f(x) = \frac{1}{x^2}$ $g = \ln(x)$ $g' = \frac{1}{x}$

b)
$$\int \underset{f}{(x+1)} \cdot \underset{g'}{\sin(2x)}\, dx = -(x+1)\frac{\cos(2x)}{2} + \frac{1}{2}\int \cos(2x)\, dx =$$
$$= -\frac{1}{2}(x+1)\cos(2x) + \frac{1}{4}\sin(2x) + c$$

6.) 8p
$$\int_0^3 \frac{5x}{x^2-3x-4}\, dx = \int_0^3 \frac{5x}{(x-4)(x+1)}\, dx = \int_0^3 \frac{4}{x-4} + \frac{1}{x+1}\, dx =$$

$$\frac{5x}{x^2-3x-4} = \frac{A}{x-4} + \frac{B}{x+1}$$

$5x = A(x+1) + B(x-4)$

$A + B = 5$ $\leftarrow$ $5B = 5$

$A - 4B = 0 \rightarrow A = 4B$

$B = 1;\ A = 4$

$$= \Big[4 \cdot \ln|x-4| + \ln|x+1|\Big]_0^3 =$$
$$= 4 \cdot \underbrace{\ln 1}_{=0} + \ln 4 - 4\ln 4 - \underbrace{\ln 1}_{=0} =$$
$$= -3\ln 4 = -6\ln 2$$