Matematika A1 vizsga GTK PSz&NG MEGOLDÁSOK 2025.01.14.

1.) 6p

$$\lim_{n\to\infty} \frac{n^2-1}{2n^2} = \lim_{n\to\infty} \frac{1}{2} - \overbrace{\frac{1}{2n^2}}^{\to 0} = \frac{1}{2}$$

felső korlát (sup): $\lim a_n = \frac{1}{2}$

$$a_n \lessgtr a_{n+1}$$

$$(n^2-1)2(n+1)^2 \gtrless (n^2+2n)\cdot 2n^2$$

$$(n^2-1)(n^2+2n+1) \gtrless 2n^4+4n^3$$

$$n^4+2n^3-2n-1 \gtrless 2n^4+4n^3$$

$$-2n-1 < n^2+2n^3 \Rightarrow$$ szig. mon. növő

alsó korlát (min.) $a_1 = 0$

2.) 7p

$$D_f: \mathbb{R} \setminus \{1,2\} \qquad x^2-3x+2 = (x-1)(x-2)$$

$$f(x) = \frac{(x-2)(x+2)}{(x-1)(x-2)} = \begin{cases} \frac{x+2}{x-1} & x \neq 2 \\ \text{n.é.} & x = 2 \end{cases}$$

$$\lim_{x\to 2\pm} f(x) = \lim_{x\to 2\pm} \frac{x+2}{x-1} = \frac{4}{1} = 4$$ megszüntethető szakadás

$$\lim_{x\to 1+} f(x) = \lim_{x\to 1+} \frac{x+2}{x-1} = \frac{3}{0+} = \infty$$

$$\lim_{x\to 1-} f(x) = \lim_{x\to 1-} \frac{x+2}{x-1} = \frac{3}{0-} = -\infty$$

pólus (másodfajú szakadás)

3.) 6p

$$T=\frac{a\cdot b}{2}=\frac{a\cdot\sqrt{4-a^2}}{2} \Rightarrow T'=\frac{1}{2}\sqrt{4-a^2}+\frac{a}{\cancel{2}}\cdot\frac{1}{2\sqrt{4-a^2}}\cdot(-\cancel{2}a)=$$

$$a^2+b^2=4$$
$$b=\sqrt{4-a^2}$$

$$=\frac{\sqrt{4-a^2}}{2}-\frac{a^2}{2\sqrt{4-a^2}}=\frac{4-2a^2}{2\sqrt{4-a^2}}$$

$$T'=0 \Rightarrow 4-2a^2=0$$
$$2=a^2$$
$$a=\sqrt{2}$$

$$b=\sqrt{4-2}=\sqrt{2}$$

$$T=\frac{\sqrt{2}\cdot\sqrt{2}}{2}=1$$

4.) 10p

$D_f:\mathbb{R}$

zérushely: $f(2)=2^3-2\cdot2^2-4\cdot2+8=0$

$$x^3-2x^2-4x+8:(x-2)=x^2-4$$
$$\underline{x^3-2x^2}$$
$$-4x+8$$
$$\underline{-4x+8}$$
$$0$$

$$f(x)=(x-2)^2(x+2)$$

zérushelyek: $\pm 2$

nincs spec. tul. $f(-x)\neq\pm f(x)$

$$\lim_{x\to\infty} x^3-2x^2-4x+8=\infty$$

$$\lim_{x\to-\infty} x^3-2x^2-4x+8=-\infty$$

vízsz. és ferde aszimptóta nincs

$$f'(x) = 3x^2 - 4x - 4 \qquad f'(x) = 0 \Rightarrow x_{1,2} = \frac{4 \pm \overbrace{\sqrt{16+4\cdot 12}}^{8}}{6} \begin{array}{l} \nearrow 2 \\ \searrow -\frac{2}{3} \end{array}$$

| | | $-2/3$ | | $2$ | |
|---|---|---|---|---|---|
| $f'$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $f$ | ↗ | lok max | ↘ | lok min | ↗ |

$$f(2) = 0$$

$$f\left(-\frac{2}{3}\right) = \left(-\frac{2}{3}\right)^3 - 2\left(-\frac{2}{3}\right)^2 - 4\left(-\frac{2}{3}\right) - 8 =$$

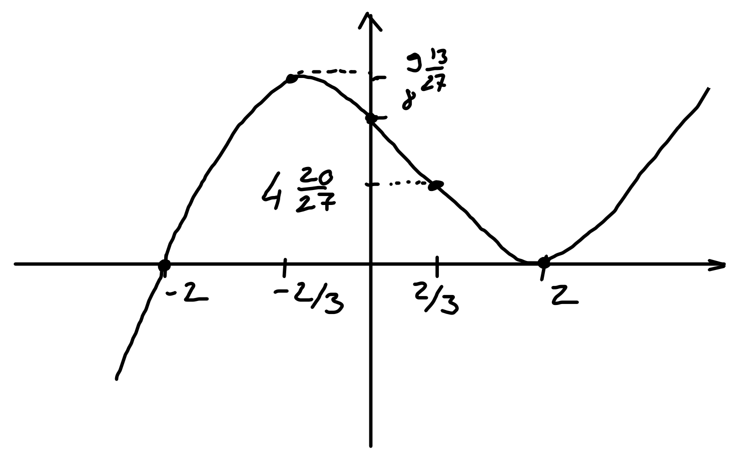
$$= -\frac{8}{27} - \frac{8}{9} + \frac{8}{3} - 8 = \frac{-8 \overbrace{-24+72}^{40}}{27} + 8 =$$

$$= 9\frac{13}{27} = \frac{256}{27}$$

$$f''(x) = 6x - 4 \qquad 6x - 4 = 0$$

$$x = \frac{2}{3}$$

| | | $2/3$ | |
|---|---|---|---|
| $f''$ | $-$ | $0$ | $+$ |
| $f$ | ⌒ | infl | ⌣ |




$$f\left(\frac{2}{3}\right) = \frac{8}{27} - \frac{8}{9} - \frac{8}{3} + 8 = \frac{\overbrace{8-24-72}^{-88}}{27} + 8 = 4\frac{20}{27} = \frac{128}{27}$$

$R_f : \mathbb{R}$

5.) 3+5p

a) $\int \sin(3x-5)\,dx = \frac{1}{3}\cdot\int \sin(3x-5)\cdot 3\,dx = \frac{-\cos(3x-5)}{3} + c$

b) $\int \frac{x+2}{x^2+2}\,dx = \frac{1}{2}\cdot\int \frac{2x}{x^2+2}\,dx + \int \frac{2}{x^2+2}\,dx = \frac{1}{2}\ln|x^2+2| + \sqrt{2}\cdot\int \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+1}\cdot\frac{1}{\sqrt{2}}\,dx =$

$= \frac{1}{2}\ln|x^2+2| + \sqrt{2}\,\operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right) + c$

6.) 8p

$\int_0^1 x\cdot e^{2x}\,dx = \left[x\cdot\frac{e^{2x}}{2}\right]_0^1 - \int_0^1 \frac{e^{2x}}{2}\,dx = \left[x\cdot\frac{e^{2x}}{2} - \frac{e^{2x}}{4}\right]_0^1 =$

$= \left(1\cdot\frac{e^2}{2} - \frac{e^2}{4}\right) - \left(0\cdot\frac{e^0}{2} - \frac{e^0}{4}\right) =$

$= \frac{1}{4}\cdot e^2 + \frac{1}{4}$