Matematika A1 vizsga GTK PSz&NG MEGOLDÁSOK 2025.01.21.

1.) 7p

$$p(x)=x^4-x^3-5x^2-3x=x\cdot(x^3-x^2-5x-3) \Rightarrow x_1=0$$

további gyökök: $\pm 1; \pm 3$ lehet

$$p(-1)=(-1)((-1)^3-(-1)^2-5(-1)-3)=-(-1-1+5-3)=0 \Rightarrow x_2=-1$$

$$\begin{array}{l}
x^3-x^2-5x-3 : (x+1) = \underbrace{x^2-2x-3} \\
-(x^3+x^2) \\
\hline
\quad -2x^2-5x-3 \\
\quad -(-2x^2-2x) \\
\hline
\qquad -3x-3 \\
\qquad -(-3x-3) \\
\hline
\qquad\qquad 0
\end{array}$$

$$x_{3,4}=\frac{2\pm\sqrt{4-4\cdot 3}}{2}=1\pm\frac{\sqrt{16}}{2}=\begin{cases}3\\-1\end{cases}$$

Szorzatalak:

$$p(x)=x\cdot(x+1)^2(x-3)$$

2.) 4+4p

a) $$\lim_{x\to 3+}\frac{x-5}{x^2-4x+3}=\lim_{x\to 3+}\frac{\overset{\to -2}{x-5}}{\underset{\searrow 0+}{(x-3)}\underset{\searrow 2}{(x-1)}}=\frac{-2}{0+\cdot 2}=-\infty$$

b) $$\lim_{x\to\infty}\frac{\ln(x)}{2\sqrt{x}}=\text{"}\frac{\infty}{\infty}\text{"} \overset{L'H}{\rightsquigarrow} \lim_{x\to\infty}\frac{\frac{1}{x}}{\cancel{2}\cdot\frac{1}{\cancel{2}\sqrt{x}}}=\lim_{x\to\infty}\frac{1}{\underset{\searrow\infty}{\sqrt{x}}}=0$$

3.) 5p

$$f(3) = \sqrt{\frac{3-1}{3+3}} = \sqrt{\frac{2}{6}} = \frac{1}{\sqrt{3}}$$ érintő:

$$f'(x) = \frac{1}{2}\sqrt{\frac{x+3}{x-1}} \cdot \frac{4}{(x+3)^2}$$

$$f'(3) = \frac{1}{2} \cdot \sqrt{\cancel{3}} \cdot \frac{\cancel{4}^{1}}{\cancel{36}_{9}} = \frac{1}{6\sqrt{3}}$$

$$y = \frac{1}{6\sqrt{3}}(x-3) + \frac{1}{\sqrt{3}}$$

$$y = \frac{1}{6\sqrt{3}}x + \frac{1}{2\sqrt{3}}$$

4.) 10p

$D_f: \mathbb{R}$; $x=0$ zérushely ; nincs spec. tul.

nincs szakadás, függ. asz. nincs

$$\lim_{x\to\infty} x \cdot e^{2x} = \infty \cdot \infty = \infty$$ (vízsz. asz. nincs)

$$\lim_{x\to\infty} \frac{x \cdot e^{2x}}{x} = \lim_{x\to\infty} e^{2x} = \infty$$ nincs ferde asz.

$$\lim_{x\to-\infty} x \cdot e^{2x} = \text{"}-\infty \cdot 0\text{"} = \lim_{x\to-\infty} \frac{\overbrace{x}^{\to -\infty}}{\underbrace{e^{-2x}}_{\to\infty}} \overset{L'H}{\rightsquigarrow} \lim_{x\to-\infty} \frac{1}{(-2)\cdot e^{-2x}} =$$

$$= \lim_{x\to-\infty} -\frac{1}{2} \cdot \underbrace{e^{2x}}_{\to 0} = 0$$ $-\infty$-ben $y=0$ vízsz. asz.

$$f'(x) = \left(x \cdot e^{2x}\right)' = e^{2x} + x \cdot 2e^{2x} = e^{2x} \cdot (1+2x)$$

$$f'(x) = 0 \implies 1 + 2x = 0$$

$$x = -\frac{1}{2}$$

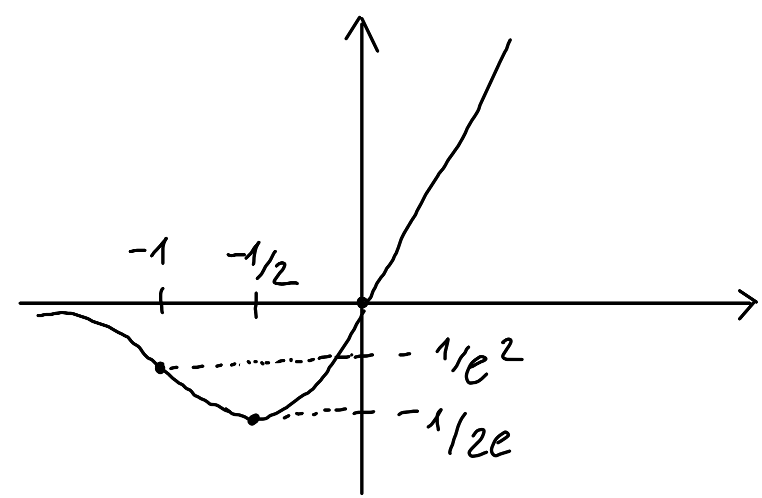
	$(-\infty, -\frac{1}{2})$	$-\frac{1}{2}$	$(-\frac{1}{2}, \infty)$
f'	$-$	0	$+$
f	$\searrow$	lok min	$\nearrow$

$$f\left(-\frac{1}{2}\right) = -\frac{1}{2} \cdot e^{2 \cdot \frac{-1}{2}} = -\frac{1}{2e}$$

$$f''(x) = \left(e^{2x}(1+2x)\right)' = e^{2x} \cdot 2 + 2 \cdot e^{2x}(1+2x) = e^{2x}(4+4x)$$

$$f''(x) = 0 \implies 4 + 4x = 0$$

$$x = -1$$



	$(-\infty, -1)$	-1	$(-1, \infty)$
f''	$-$	0	$+$
f	$\frown$	infl	$\smile$

$$f(-1) = (-1)\, e^{2 \cdot (-1)} = \frac{-1}{e^2}$$

$$R_f : \left[-\frac{1}{2e}, \infty\right)$$

5.) 4+4p

a) $\int \frac{\sqrt{x} + 2x + 3}{x^2}\,dx = \int \frac{1}{x^{3/2}} + \frac{2}{x} + \frac{3}{x^2}\,dx = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 2\ln|x| + \frac{3x^{-1}}{-1} + C =$

$= \frac{-2}{\sqrt{x}} + 2\ln|x| - \frac{3}{x} + C$

b) $\int x \cdot sh(2x)\,dx = x\frac{ch(2x)}{2} - \int \frac{ch(2x)}{2}\,dx = x \cdot \frac{ch(2x)}{2} - \frac{sh(2x)}{4} + C$

6.) 7p

$\int_0^1 \underset{\downarrow\ g'(x)}{2x} \cdot \underbrace{(x^2+1)^4}_{f(g(x))}\,dx = \left[\frac{(x^2+1)^5}{5}\right]_0^1 = \frac{(1+1)^5}{5} - \frac{1^5}{5} =$

$f(x) = x^4$

$\int f(x)\,dx = \frac{x^5}{5}$

$\frac{2^5}{5} - \frac{1}{5} = \frac{32-1}{5} = \frac{31}{5}$