2. zh pótlása – MEGOLDÁSOK 2023. dec. 13.

1) a) $\lim\limits_{x\to 0+} \dfrac{\overbrace{\sqrt{1-x^2}}^{\to 1}-1}{1-\underbrace{\sqrt{1-x}}_{\to 1}} = \text{"}\dfrac{0}{0}\text{"} = \lim\limits_{x\to 0+} \dfrac{(\sqrt{1-x^2}-1)(\sqrt{1-x^2}+1)}{(1-\sqrt{1-x})(1+\sqrt{1-x})} \cdot \dfrac{1+\sqrt{1-x}}{\sqrt{1-x^2}+1} =$

$= \lim\limits_{x\to 0+} \dfrac{1-x^2-1^2}{1^2-(1-x)} \cdot \dfrac{1+\sqrt{1-x}}{\sqrt{1-x^2}+1} = \lim\limits_{x\to 0+} \underbrace{\dfrac{-x^2}{x}}_{=-x\to 0} \cdot \underbrace{\dfrac{\overbrace{1+\sqrt{1-x}}^{\to 1}}{\sqrt{1-x^2}+1}}_{\to 1} = 0 \cdot \dfrac{1+1}{1+1} = \underline{\underline{0}}$

Vagy L'Hopitállal

$\hookrightarrow \lim\limits_{x\to 0+} \dfrac{\dfrac{1}{2\sqrt{1-x^2}} \cdot -2x}{\dfrac{-1}{2\sqrt{1-x}} \cdot (-1)} = \lim\limits_{x\to 0+} \underbrace{-x}_{\to 0} \cdot \dfrac{\overbrace{2\sqrt{1-x}}^{\to 2\cdot 1}}{\underbrace{\sqrt{1-x^2}}_{\to 1}} = 0 \cdot \dfrac{2}{1} = \underline{\underline{0}}$

b) $\lim\limits_{x\to\infty} \dfrac{x^2}{e^{2x}+1} = \text{"}\dfrac{\infty}{\infty}\text{"} \underset{L'H}{\rightsquigarrow} \lim\limits_{x\to\infty} \dfrac{2x}{2e^{2x}} = \text{"}\dfrac{\infty}{\infty}\text{"} \underset{L'H}{\rightsquigarrow} \lim\limits_{x\to\infty} \dfrac{2}{4\cdot e^{2x}} = \dfrac{2}{\infty} = \underline{\underline{0}}$

2.) $f(1) = (1^2+1)\ln(2\cdot 1-1) = 2\ln(1) = 2\cdot 0 = 0$

$f'(x) = (2x+1)\ln(2x-1) + (x^2+x)\dfrac{1}{2x-1}\cdot 2$

$f'(1) = (2\cdot 1+1)\ln(1) + (1^2+1)\dfrac{1}{2\cdot 1-1}\cdot 2 = 3\cdot\ln(1) + \dfrac{2\cdot 2}{1} = 0+4 = 4$

érintő: $y = 4(x-1)+0 = 4x-4$

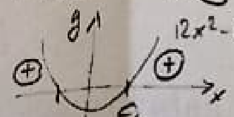
3.) $f'(x) = 4x^3-8x$

$f''(x) = 12x^2-8 \nearrow \quad 12x^2-8=0$

$x^2 = \dfrac{2}{3}$

$x_{1,2} = \pm\sqrt{\dfrac{2}{3}}$

$D_f = \mathbb{R}$



x	$(-\infty,-\sqrt{\frac{2}{3}})$	$-\sqrt{\frac{2}{3}}$	$(-\sqrt{\frac{2}{3}},\sqrt{\frac{2}{3}})$	$\sqrt{\frac{2}{3}}$	$(\sqrt{\frac{2}{3}},\infty)$
f''	+	0	−	0	+
f	$\cup$	Infl	$\cap$	Infl	$\cup$

4.) $D_f = \mathbb{R}\setminus\{1\}$

$\lim\limits_{x\to 1+} \dfrac{\overbrace{x(2x-1)}^{\to 1}}{\underbrace{x-1}_{\to 0+}} = \dfrac{1}{0+} = \infty \quad \lim\limits_{x\to 1-} \dfrac{\overbrace{x(2x-1)}^{\to 1}}{\underbrace{x-1}_{\to 0-}} = \dfrac{1}{0-} = -\infty$ az $x=1$ függőleges aszimptota

$\lim\limits_{\substack{x\to\infty \\ (-\infty)}} \dfrac{2x^2-x}{x-1} = \text{"}\dfrac{\infty}{(-)\infty}\text{"} = \lim\limits_{\substack{x\to\infty \\ (-\infty)}} \dfrac{2x-1}{1-\frac{1}{x}} = \dfrac{\infty}{1} = \underset{(-\infty)}{\infty}$ vízszintes aszimptota nincs

$\lim\limits_{\substack{x\to\infty \\ (-\infty)}} \dfrac{f(x)}{x} = \lim\limits_{\substack{x\to\infty \\ (-\infty)}} \dfrac{2x-1}{x-1} = 2 = a \quad \lim\limits_{\substack{x\to\infty \\ (-\infty)}} f(x)-2x = \lim\limits_{\substack{x\to\infty \\ (-\infty)}} \dfrac{2x^2-x}{x-1} - 2x =$

$= \lim\limits_{\substack{x\to\infty \\ (-\infty)}} \dfrac{\cancel{2x^2}-x-\cancel{2x^2}+2x}{x-1} = \lim\limits_{\substack{x\to\infty \\ (-\infty)}} \dfrac{x}{x-1} = 1 = b$

∞-ben $y = 2x+1$ ferde aszimptota

($-\infty$-ben ugyanez adódik)