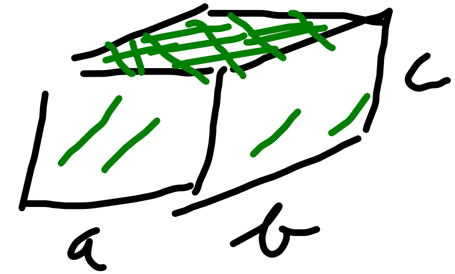
Konzultáció P0 kurzus 2024.06.03.

2023. 06.13 vizsga / 4. fel.



Téglatest $V = a \cdot b \cdot c = 1\,m^3$

$F = 4a \cdot b + 2ac + 2bc$

$f(a,b) = 4ab + 2\cdot\cancel{a}\frac{1}{\cancel{a}b} + 2\cancel{b}\frac{1}{a\cancel{b}} = \underbrace{4ab + \frac{2}{b} + \frac{2}{a}}_{\text{minimum?}}$

$a, b, c > 0$

1 Stac. pontok

$f'_a(a,b) = 0 = 4b \cdot 1 + 0 + 2(-1)\cdot a^{-2} = 4b - \frac{2}{a^2}$

$f'_b(a,b) = 0 = 4a \cdot 1 + 2\cdot(-1)\cdot b^{-2} + 0 = 4a - \frac{2}{b^2}$

$$4b - \frac{2}{a^2} = 0$$
$$4a - \frac{2}{b^2} = 0$$

$$b = \frac{2}{a^2} \cdot \frac{1}{4} = \frac{1}{2a^2}$$

$$4a - \frac{2}{\left(\frac{1}{2a^2}\right)^2} = 0$$

$$4a - 2 \cdot \frac{1}{1/4a^4} = 0$$

$$4a - 2 \cdot 4a^4 = 0 \quad /:4$$

$$a - 2a^4 = 0$$

$$a(1 - 2a^3) = 0$$

$a = 0$

nem jó $a > 0$

$$1 - 2a^3 = 0$$

$$a^3 = \frac{1}{2}$$

$$b = \frac{1}{2 \cdot \left(2^{-\frac{1}{3}}\right)^2} = \frac{1}{\sqrt[3]{2}} \leftarrow a = \frac{1}{\sqrt[3]{2}} = 2^{-\frac{1}{3}}$$

$$a = \frac{1}{\sqrt[3]{2}} ; b = \frac{1}{\sqrt[3]{2}} \Rightarrow c = \frac{1}{ab} = \frac{1}{\frac{1}{2^{2/3}}} = \sqrt[3]{4}$$

$$a = b = \frac{1}{\sqrt[3]{2}} \qquad P\left(\frac{1}{\sqrt[3]{2}} ; \frac{1}{\sqrt[3]{2}}\right)$$

(2) P minimum hely?

$$f''_{aa} = \left(4b \cdot \frac{2}{a^2}\right)'_a = 0 - 2 \cdot (-2) \cdot \frac{1}{a^3} = \frac{4}{a^3}$$

$$f''_{ab} = f''_{ba} = \left(4b - \frac{2}{a^2}\right)'_b = 4 - 0$$

$$f''_{bb} = \left(4a - \frac{2}{b^2}\right)'_b = 0 - 2(-2) \cdot \frac{1}{b^3} = \frac{4}{b^3}$$

$$\det\left(\text{Hesse}\left(1/\sqrt[3]{2}, 1/\sqrt[3]{2}\right)\right) = \left. \begin{vmatrix} 4/a^3 & 4 \\ 4 & 4/b^3 \end{vmatrix} \right|_{(1/\sqrt[3]{2} ; 1/\sqrt[3]{2})} =$$

$$= \begin{vmatrix} 8 & 4 \\ 4 & 8 \end{vmatrix} = 8^2 - 4^2 = 64 - 16 > 0$$

van szélsőérték!

$$f''_{aa}\left(\frac{1}{\sqrt[3]{2}} ; \frac{1}{\sqrt[3]{2}}\right) = 8 > 0 \Rightarrow \text{MINIMUM}$$

11fs / F4 Geom. sorösszeg

$$\sum_{n=0}^{\infty} \frac{2+3^{n-1}}{5^n}$$

$$\sum_{n=0}^{\infty} a\cdot q^n = a\cdot 1 + a\cdot q + a\cdot q^2 + \dots$$

ha $|q|<1 \Rightarrow \displaystyle\sum_{n=0}^{\infty} a\cdot q^n = a\cdot\frac{1}{1-q}$

$\parallel$

$$\sum_{n=0}^{\infty} \frac{2}{5^n}+\frac{3^{n-1}}{5^n} \overset{?}{=} \sum_{n=0}^{\infty}\frac{2}{5^n} + \sum_{n=0}^{\infty}\frac{3^{n-1}}{5^n}$$

$$\sum_{n=0}^{\infty} \underbrace{2}_{a}\cdot\underbrace{\left(\frac{1}{5}\right)}_{=q}^{n} = 2\cdot\frac{1}{1-\frac{1}{5}} = \cancel{2}\cdot\frac{5}{\cancel{4}} = \frac{5}{2}$$

$\left|\frac{1}{5}\right|<1$

$$\sum_{n=0}^{\infty}\frac{3^{n-1}}{5^n} = \sum_{n=0}^{\infty}\frac{1}{3}\cdot\frac{3^n}{5^n} = \sum_{n=0}^{\infty}\overset{a}{\frac{1}{3}}\cdot\overset{q<1}{\left(\frac{3}{5}\right)}^{n} = \frac{1}{3}\cdot\frac{1}{1-\frac{3}{5}} = \frac{1}{3}\cdot\frac{5}{2} = \frac{5}{6}$$

$$\sum_{n=0}^{\infty}\frac{2+3^{n-1}}{5^n} = \frac{5}{2}+\frac{5}{6} = \frac{15}{6}+\frac{5}{6} = \frac{20}{6}$$

10FS / F4

$$f(x,y) = \frac{x \cdot y}{27} + \frac{1}{x} + \frac{1}{y} \longrightarrow \text{súbsčíte?}$$

① $$f'_x(x,y) = \frac{y}{27} \cdot 1 + \frac{-1}{x^2} + 0 = \frac{y}{27} - \frac{1}{x^2} = 0$$

$$f'_y(x,y) = \frac{x}{27} \cdot 1 + 0 - \frac{1}{y^2} = \frac{x}{27} - \frac{1}{y^2} = 0$$

$$\Rightarrow \frac{y}{27} - \frac{1}{x^2} = 0$$

$$y = \frac{27}{x^2} \longrightarrow \frac{x}{27} - \frac{1}{\left(\frac{27}{x^2}\right)^2} = 0$$

$$\frac{x}{\cancel{27}} - \frac{x^4}{27^{\cancel{2}}} = 0$$

$$27x - x^4 = 0$$

$$x(27 - x^3) = 0$$

$$x \cdot (27 - x^3) = 0$$

$x = 0$

$y = \frac{27}{x^2}$???

$\Rightarrow$ f nem értelmezett ha $x=0$!

$\Rightarrow$ nem ad stac. pontot

$27 - x^3 = 0$

$x = 3$

$y = \frac{27}{3^2} = 3$

$P(3,3) \Rightarrow$ stac. pont

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újra

$$\left.\begin{aligned} \frac{y}{27} - \frac{1}{x^2} = 0 \\ \frac{x}{27} - \frac{1}{y^2} = 0 \end{aligned}\right\} \begin{aligned} &\to y \cdot x^2 - 27 = 0 \to yx^2 = 27 \\ &\to x \cdot y^2 - 27 = 0 \to xy^2 = 27 \end{aligned}$$

$yx^2 = xy^2$ $x, y \neq 0$ $\to$ $\boxed{x = y}$ $y^3 = 27$ $\downarrow$ $x = y = 3$

② Hesse : $f''_{xx} = \left(\frac{y}{27} - \frac{1}{x^2}\right)'_x = \left(-x^{-2}\right)' = 2 \cdot \frac{1}{x^3}$

$f''_{xy} = f''_{yx} = \left(\frac{y}{27} - \frac{1}{x^2}\right)'_y = \frac{1}{27}$

$f''_{yy} = \left(\frac{x}{27} - \frac{1}{y^2}\right)'_y = 0 + 2 \cdot \frac{1}{y^3}$

$\det\left(\text{Hesse}(x,y)\right) = \begin{vmatrix} \frac{2}{x^3} & \frac{1}{27} \\ \frac{1}{27} & \frac{2}{y^3} \end{vmatrix} = \frac{4}{x^3 \cdot y^3} - \frac{1}{27^2}$

$P(3,3) \Rightarrow \det\left(\text{Hesse}(3,3)\right) = \begin{vmatrix} \frac{2}{27} & \frac{1}{27} \\ \frac{1}{27} & \frac{2}{27} \end{vmatrix} =$

$= \frac{4}{27^2} - \frac{1}{27^2} = \frac{\overbrace{4-1}^{=3>0}}{\underbrace{27^2}_{>0}} > 0 \Rightarrow$ van szélsőérték!

$f''_{xx}(3,3) = f''_{yy}(3,3) = \frac{2}{27} > 0 \Rightarrow$ minimum hely!

$f(3,3) = \frac{3 \cdot 3}{27} + \frac{1}{3} + \frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

12fs/F4 $f(x) = x \cdot \sin(2x)$

$\hookrightarrow \sum_{n=0}^{\infty} \frac{\boxed{f^{(n)}(0)}}{n!} x^n$ $x_0 = 0$

körüli Taylor-sor

$\sin(x) = \sum$ .... vizsgán nem lesz!

$\sin(2x) \overset{x \to 2x}{=} \sum \dots$

23.06.07./5f)

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ -> Leibniz

- alt.
- $\lim a_n = 0$
- mon. csö. $|a_n|$

$\Rightarrow$ konvergens a sor

$\Rightarrow$ Vajon absz. konv.-e? $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$?

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$$ pozitív tagú!

$$\sim \sqrt{n} = n^{1/2}$$

$$|a_n| = \frac{1}{\sqrt{n+1}} > \frac{1}{\sqrt{n+n}} = \frac{1}{\sqrt{2n}} = \underbrace{\left(\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{n}}\right)}_{= b_n > 0}$$

$$\sum_{n=1}^{\infty} b_n = \frac{1}{\sqrt{2}} \cdot \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \rightarrow \text{divergens } (p = 1/2)$$

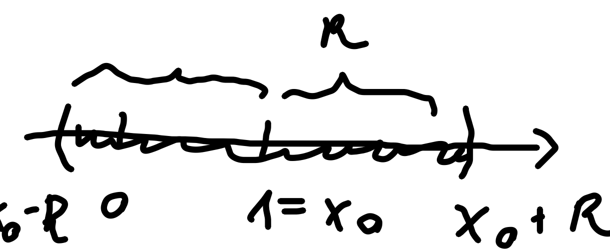
$\Rightarrow$ minoráns krit. alapján $\Rightarrow$ $\sum |a_n|$ divergens

$\Rightarrow$ eredeti sor feltételesen konvergens

Minta vizsga 6fel:

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2\cdot 2^n} = \sum_{n=1}^{\infty} \underbrace{\frac{1}{n^2\cdot 2^n}}_{a_n} (x-1)^n$$

$$(x-x_0)^n \Downarrow x_0 = 1$$



$R = ?$

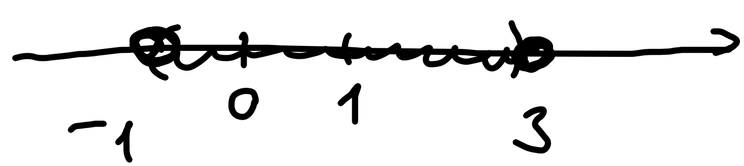
$$\left|\sqrt[n]{a_n}\right|$$

$$\frac{1}{R} = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{1}{(n+1)^2\cdot 2^{n+1}}\cdot\frac{n^2\cdot \cancel{2^n}}{1} =$$

$$\underbrace{\lim\left|\frac{a_{n+1}}{a_n}\right|}\cdot|x-1| < 1$$

$$= \lim_{n\to\infty} \underbrace{\frac{n^2}{(n+1)^2}}_{\to 1}\cdot\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{R} = \frac{1}{2} \rightarrow R = 2$$



$x \in (-1, 3) \Rightarrow$ akkor a sor konvergens

$x = -1$ konv.!

$$\sum_{n=1}^{\infty} \frac{(-1-1)^n}{n^2 \cdot 2^n} = \sum_{n=1}^{\infty} \frac{(-2)^n}{n^2 \cdot 2^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \to \text{Leibniz-sor}$$

alternál

$-\lim \frac{(-1)^n}{n^2} = 0$

$-\frac{1}{n^2}$ mon. csökk.

$x = 3$

$$\sum_{n=1}^{\infty} \frac{(3-1)^n}{n^2 \cdot 2^n} = \sum_{n=1}^{\infty} \frac{1}{n^2} \quad 2 = p$$

$p > 1$ tehát konv.

$\Rightarrow x \in [-1, 3]$-on konvergens a hatványsor

$$\frac{1}{R} = \lim_{n \to \infty} \sqrt[n]{a_n} = \lim_{n \to \infty} \sqrt[n]{\frac{1}{n^2 \cdot 2^n}} = \lim_{n \to \infty} \frac{1}{\underbrace{(\sqrt[n]{n})^2}_{\to 1^2} \cdot 2} = \frac{1}{2}$$

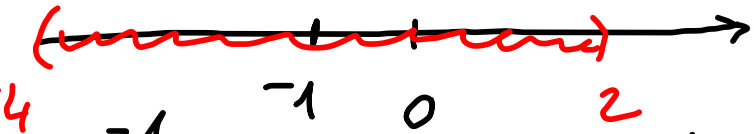
$$R = \frac{1}{1/2} = 2$$

12fs FZ

$$\sum_{n=1}^{\infty} \frac{n \cdot (x+1)^n}{3^{n+1}} = \sum_{n=1}^{\infty} \underbrace{\frac{n}{3^{n+1}}}_{a_n} \underbrace{\left(x-(-1)\right)^n}_{(x-x_0)^n}$$

$x_0 = -1$

$$\frac{1}{R} = \lim_{n\to\infty} \sqrt[n]{|a_n|} =$$



$$= \lim_{n\to\infty} \sqrt[n]{\frac{n}{3^{n+1}}} = \lim_{n\to\infty} \frac{\overbrace{\sqrt[n]{n}}^{\to 1}}{\sqrt[n]{3 \cdot 3^n}} = \lim_{n\to\infty} \frac{\overbrace{\sqrt[n]{n}}^{\to 1}}{\underbrace{\sqrt[n]{3}}_{\to 1} \cdot 3} = \frac{1}{1 \cdot 3} = \frac{1}{3}$$

$R = 3$

$$\frac{1}{R} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\frac{n+1}{3^{n+1+1}}}{\frac{n}{3^{n+1}}} = \lim_{n\to\infty} \overbrace{\frac{n+1}{\cancel{3^{n+2}}} \cdot \frac{\cancel{3^{n+1}}}{n}}^{\to 1} \to \frac{1}{3} = \frac{1}{3}$$

$x=-4 \quad \sum_{n=1}^{\infty} \frac{n}{3^{n+1}} \cdot (-4+1)^n = \sum_{n=1}^{\infty} \frac{n}{3^{n+1}} \cdot (-3)^n = \sum_{n=1}^{\infty} \frac{n}{3} \cdot (-1)^n =$

$= -\frac{1}{3} + \frac{2}{3} - \frac{3}{3} + \frac{4}{3} \dots\dots$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^n \cdot \frac{n}{3} \neq 0$ ált. tag nem tart a 0-hoz

$\Rightarrow$ a sor divergens

$x=2 \quad \sum_{n=1}^{\infty} \frac{n}{3^{n+1}} \cdot (2+1)^n = \sum_{n=1}^{\infty} \frac{n}{3} = \frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \dots$

$\lim_{n\to\infty} \frac{n}{3} \neq 0 \Rightarrow$ div.

$x \in (-4, 2)$ lesz a konv. a hatványsor