Konzultáció P0 kurzus   2024.06.10.

Minta vizsga F5

$$f(x,y) = \frac{\sqrt{y}}{x+2y} \qquad f'_x(x,y) = \left(\frac{\sqrt{y}}{x+2y}\right)'_x = *$$

$$* = \sqrt{y}\cdot\left(\frac{1}{x+2y}\right)' = \sqrt{y}\cdot\left(\left(x+2y\right)^{-1}\right)' = \sqrt{y}\cdot(-1)\,(x+2y)^{-2}\cdot 1 =$$

$$\left(\frac{1}{x+c}\right)' = (-1)(x+c)^{-2}\cdot 1 \qquad = \frac{-\sqrt{y}}{(x+2y)^2}$$

$$f'_y(x,y) = \left(\frac{\sqrt{y}}{x+2y}\right)'_y = \frac{\frac{1}{2}\cdot y^{-\frac{1}{2}}\cdot(x+2y) - \sqrt{y}\cdot 2}{(x+2y)^2} =$$

$$= \frac{\frac{1}{2\sqrt{y}}\cdot(x+2y) - 2\sqrt{y}}{(x+2y)^2}\cdot\frac{2\sqrt{y}}{2\sqrt{y}} = \frac{x+2y-4y}{2\sqrt{y}\cdot(x+2y)^2} =$$

$$= \frac{x-2y}{2\sqrt{y}\cdot(x+2y)^2}$$

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \qquad \det(\underline{\underline{A}} - \lambda \underline{\underline{E}}) = \begin{vmatrix} 1-\lambda & -1 & 1 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} =$$

$$= (1-\lambda) \begin{vmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)(3-\lambda)$$

$$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = 3$$

$\lambda_1 = 1$

$$(\underline{\underline{A}} - 1\underline{\underline{E}}) \cdot \underline{s} = \underline{0} \Rightarrow \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \quad 0 \cdot x - y + z = 0 \longrightarrow -y + z = 0 \ \&$$

$$0 \cdot x + y + 0 z = 0 \longrightarrow y = 0$$

$$0 \cdot x + 0 \cdot y + 2z = 0 \longrightarrow z = 0$$

$$x \in \mathbb{R}$$

$$\underline{s} = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} ; \ x \in \mathbb{R} \setminus \{0\}$$

$\lambda_2 = 2$

$$(\underline{\underline{A}} - 2\underline{\underline{E}})\underline{s} = \underline{0} \qquad \begin{pmatrix} -1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \left.\begin{array}{r} -x - y + z = 0 \\ 0 \cdot x + 0 \cdot y + 0 \cdot z = 0 \\ 0 \cdot x + 0 \cdot y + 1 \cdot z = 0 \end{array}\right\} \quad \begin{array}{l} -x - y + z = 0 \\ 0 = 0 \ \& \\ z = 0 \end{array} \Rightarrow$$

$$\Rightarrow \begin{array}{l} -x - y = 0 \\ z = 0 \end{array}$$

$x = -y$

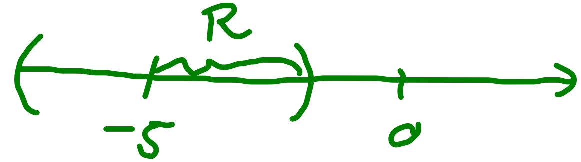
$$\underline{s} = \begin{pmatrix} -y \\ y \\ 0 \end{pmatrix} \quad y \in \mathbb{R} \setminus \{0\}$$

$y = -x$

$$\underline{\tilde{s}} = \begin{pmatrix} x \\ -x \\ 0 \end{pmatrix} \quad x \in \mathbb{R} \setminus \{0\}$$

Konv. tartomány $\sum_{n=0}^{\infty} a_n \cdot (x-x_0)^n$

$$\sum_{n=\cancel{0}\,1}^{\infty} \underbrace{\frac{1}{n^3+n}}_{=a_n}(x+5)^n \Rightarrow x_0=-5 \rightarrow$$




$$\frac{1}{R} = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\sqrt[n]{|a_n|} \quad |x-x_0| < \frac{1}{\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|}$$

VAGY

$$\lim_{n\to\infty}\left|\frac{a_{n+1}\cdot\cancel{(x-x_0)}^{\cancel{n}+1}}{a_n\cdot\cancel{(x-x_0)^n}}\right| = |x-x_0|\cdot\lim\left|\frac{a_{n+1}}{a_n}\right| < 1$$

itt:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{\frac{1}{(n+1)^3+n+1}}{\frac{1}{n^3+n}} = \lim_{n\to\infty}\frac{n^3+n}{n^3+3n^2+4n+2} =$$

$$= \lim_{n\to\infty} \frac{1 + \underbrace{\frac{1}{n^2}}_{\to 0}}{1 + \underbrace{\frac{3}{n}}_{\to 0} + \underbrace{\frac{4}{n^2}}_{\to 0} + \underbrace{\frac{2}{n^3}}_{\to 0}} = 1 = \frac{1}{R} \quad R = 1$$

$-5$ középpont, 1 sugarú intervallum $[?(\underbrace{-6, -4}_{\text{konv.}})?]$

$x = -6$

$$\sum_{n=1}^{\infty} \frac{1}{n^3+n} \cdot (-6+5)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+n}$$

- alternál
- $\lim a_n = 0$
- $|a_{n+1}| < |a_n|$

$$\frac{1}{(n+1)^3+n+1} < \frac{1}{n^3+n}$$

Leibniz $\to$ konv.

$x = -4$

$$\sum_{n=1}^{\infty} \frac{1}{n^3+n} (-4+5)^n = \sum_{n=1}^{\infty} \frac{1}{n^3+n}$$

$$\frac{1}{n^3+n} < \frac{1}{n^3} \quad \sum \frac{1}{n^3} \text{ konv.}$$

majoráns sor

$\sum \frac{1}{n^p}$ $\quad p > 1$ konv.
$\quad p \leq 1$ div

$\Rightarrow$ Konv. intervallum $[-6, -4]$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} = \frac{-1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!} - \dots$$

- vált. előjel ✓
- $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} (-1)^n \underbrace{\frac{1}{(2n+1)!}}_{\to\infty} = 0$ ✓
- $|a_{n+1}| \overset{?}{<} |a_n|$ mon. csökk.

Leibniz

$\Rightarrow$ konvergens

$$\frac{1}{(2(n+1)+1)!} \overset{?}{<} \frac{1}{(2n+1)!}$$

$$\frac{1}{(2n+3)!} \overset{?}{<} \frac{1}{(2n+1)!}$$

IGEN: $(2n+3)! > (2n+1)!$

$(2n+3)! = 1\cdot 2\cdot 3\dots (2n+1)\cdot(2n+2)\cdot(2n+3)$

$> 1\cdot 2\cdot 3\dots (2n+1)$

$$\Rightarrow \sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \left| \frac{(-1)^n}{(2n+1)!} \right| = \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} = \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \dots$$

pozitív tag

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{\frac{1}{(2(n+1)+1)!}}{\frac{1}{(2n+1)!}} = \lim_{n \to \infty} \frac{\frac{1}{(2n+3)!}}{\frac{1}{(2n+1)!}} =$$

$$\lim_{n \to \infty} \frac{(2n+1)!}{(2n+3)!} = \lim_{n \to \infty} \frac{\cancel{1} \cdot \cancel{2} \cdots \cancel{(2n+1)} \cdot 1}{\cancel{1} \cdot \cancel{2} \cdots \cancel{(2n+1)} \underbrace{(2n+2)(2n+3)}} =$$

$$= \lim_{n \to \infty} \frac{1}{(2n+2)(2n+3) \to \infty} = 0 < 1 \Rightarrow$$ hányados krit. miatt az abszolút sor is konvergens

$$\Rightarrow \sum a_n = \sum \frac{(-1)^n}{(2n+1)!}$$ eredeti sor abszolút konvergens