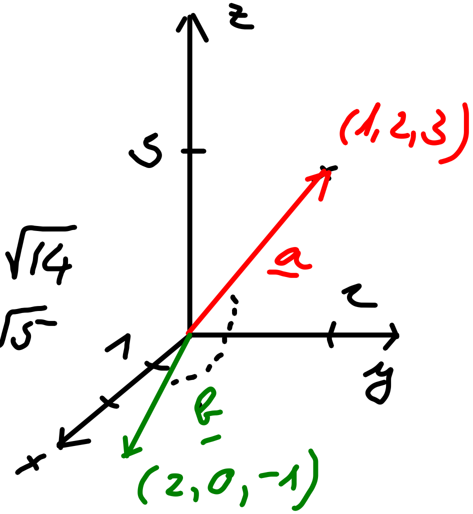
Konzultáció P0 kurzus   2024.06.24.



$\underline{v} \in \mathbb{R}^3 \quad \underline{v}\,(x, y, z)$

$\underline{a}\,(1,2,3) \;\longrightarrow\; |\underline{a}| = \sqrt{1^2+4+9} = \sqrt{14}$

$\underline{b}\,(2,0,-1) \qquad\qquad |\underline{b}| = \sqrt{4+1} = \sqrt{5}$

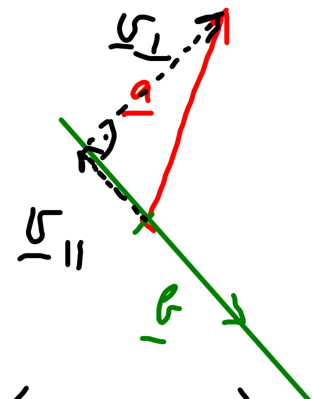
1.) Skalár szorzás

$\langle \underline{a}, \underline{b} \rangle = 1\cdot 2 + 2\cdot 0 + 3\cdot(-1) = -1$

$\parallel$

$|\underline{a}|\,|\underline{b}| \cdot \cos \measuredangle(\underline{a}, \underline{b})$

$\cos \measuredangle = \dfrac{\langle \underline{a}, \underline{b} \rangle}{|\underline{a}|\,|\underline{b}|} = \dfrac{-1}{\sqrt{14}\cdot\sqrt{5}} \quad \ldots\ldots$



2.) $\underline{a} = \underline{v}_{\parallel} + \underline{v}_{\perp} \qquad \underline{v}_{\parallel} \parallel \underline{b}$

$\qquad\qquad\qquad\qquad\qquad \underline{v}_{\perp} \perp \underline{b}$

$\underline{v}_{\parallel} = \dfrac{\langle \underline{b}, \underline{a} \rangle}{\underbrace{\langle \underline{b}, \underline{b} \rangle}_{= |\underline{b}|^2}} \cdot \underline{b} = \dfrac{-1}{5} \cdot (2, 0, -1) = \left(-\dfrac{2}{5}, 0, \dfrac{1}{5}\right)$

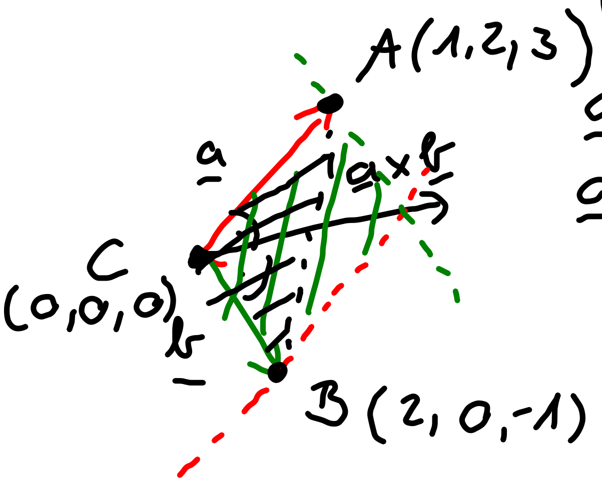
$\underline{v}_{\perp} = \underline{a} - \underline{v}_{\parallel} = (1,2,3) - \left(\dfrac{-1}{5}\right)(2,0,-1) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -2/5 \\ 0 \\ 1/5 \end{pmatrix} = \begin{pmatrix} 7/5 \\ 2 \\ 14/5 \end{pmatrix}$

3.) vektoriális szorzás

$$(1\ 0\ 0) = \underline{i}$$

$$\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 2 & 3 \\ 2 & 0 & -1 \end{vmatrix} = (2\cdot(-1) - 0\cdot 3)\,\underline{i} - (1\cdot(-1) - 2\cdot 3)\,\underline{j} + (1\cdot 0 - 2\cdot 2)\,\underline{k} =$$

$$= -2\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + 7\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} - 4\cdot\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}-2\\7\\-4\end{pmatrix}$$



$\underline{a}\times\underline{b} \perp \underline{a}$

$\underline{a}\times\underline{b} \perp \underline{b}$

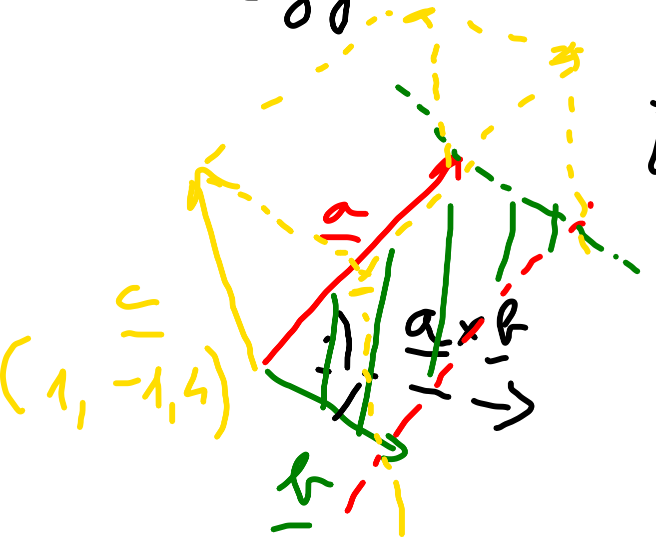
$$T_{\underline{a}\underline{b}\text{ paralelogram}} = |(-2,7,-4)| = |\underline{a}|\cdot|\underline{b}|\cdot\sin\measuredangle = \sqrt{4+49+16} = \sqrt{69}$$

$\underline{a} = \overrightarrow{CA}$

$\underline{b} = \overrightarrow{CB}$

$$T_{ABC\triangle} = \frac{|\underline{a}\times\underline{b}|}{2}$$

Vegyes szorzat



paralelepipedon

$$[\underline{a}\ \underline{b}\ \underline{c}] = \langle \underline{a}\times\underline{b}, \underline{c}\rangle =$$

$$\langle(-2,7,-4),(1,-1,4)\rangle =$$

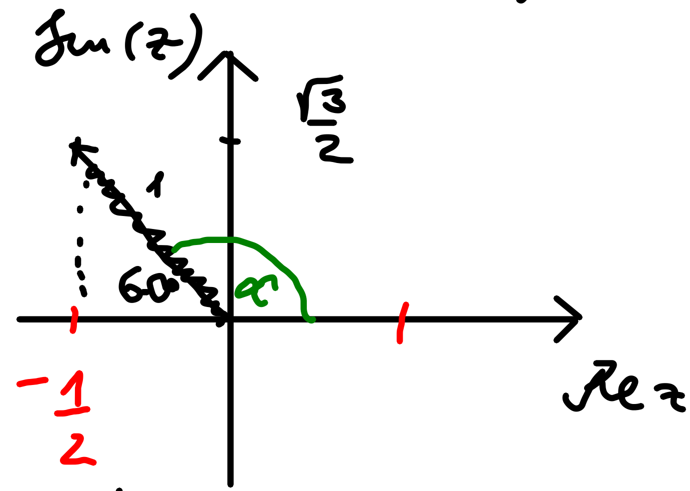
$$= (-2)\cdot 1 + 7\cdot(-1) + (-4)\cdot 4 =$$

$$= -2 - 7 - 16 = -25 = \begin{vmatrix}1&-1&4\\1&2&3\\2&0&-1\end{vmatrix} = \begin{vmatrix}1&2&3\\2&0&-1\\1&-1&4\end{vmatrix}$$

$$V_p = \left|[\underline{a},\underline{b},\underline{c}]\right| = |-25| = 25$$

Komplex számok trigonometrikus alakja

$z = a + ib = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$



$r = |z| = \sqrt{a^2+b^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{1} = 1$

$\alpha = ?$

$z = r\cdot(\cos\alpha + i\sin\alpha) = \underbrace{\overset{1}{\overset{\parallel}{r}}\cdot\cos\alpha}_{a=-\frac{1}{2}} + i\,\underbrace{\overset{1}{\overset{\parallel}{r}}\cdot\sin\alpha}_{b=\frac{\sqrt{3}}{2}}$

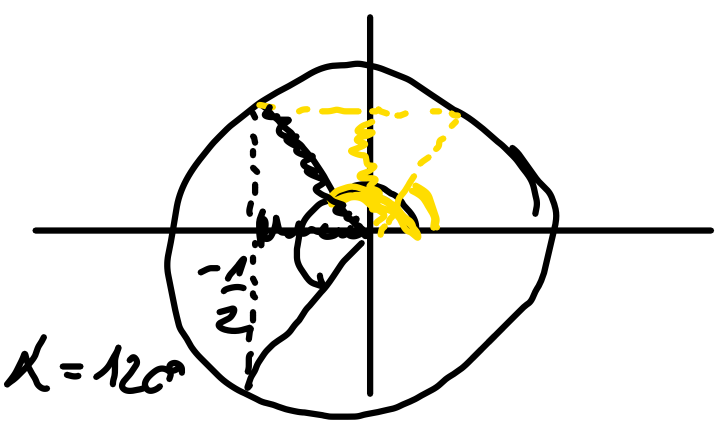
$\cos\alpha = \frac{a}{r} = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$

$\alpha = 120°$ v. $240°$

$\sin\alpha = \frac{b}{r} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$

$\alpha = 120°$ v. $60°$

$\alpha = 120°$



$$z = 1 \cdot (\cos 120^\circ + i \sin 120^\circ)$$

Hatványsor $\sum_{n=0}^{\infty} a_n \cdot (x-x_0)^n =$

$= \underbrace{a_0}_{\in \mathbb{R}} \cdot 1 + \underbrace{a_1}_{\in \mathbb{R}} \cdot (x - \underbrace{x_0}_{\text{középpont}}) + a_2 (x-x_0)^2 + \ldots\ldots$

$\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \underbrace{\frac{1}{\sqrt{n}}}_{\to a_n} \cdot (x - \underbrace{(-2)}_{x_0 = -2})^n = \frac{1}{\sqrt{1}} \cdot (x+2)^1 + \frac{1}{\sqrt{2}} \cdot (x+2)^2 + \ldots$

Mely x-re konvergens?

$|a_n \cdot (x-x_0)^n| \xrightarrow{\text{hányados krit}} \lim_{n\to\infty} \frac{|a_{n+1} \cdot (x-x_0)^{\cancel{n+1}}|}{|a_n \cdot \cancel{(x-x_0)^n}|} < 1$

$|x-x_0| \cdot \underbrace{\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|}_{H} < 1 \qquad |x-x_0| < \frac{1}{H} = R$ konv. sug.

$$H=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{1}{\sqrt{n+1}}}{\frac{1}{\sqrt{n}}}\right| = \lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+1}} = \text{"}\frac{\infty}{\infty}\text{"}$$

$$\left(\left|\sqrt[n]{a_n}\right|\right)$$

$$=\lim_{n\to\infty}\frac{\sqrt{1}}{\sqrt{1+\frac{1}{n}}} = 1 \quad \rightarrow \quad R=\frac{1}{H}=1$$

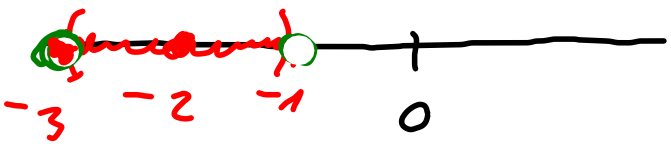
$$\sqrt{1}\to 1 \qquad \sqrt{1+\tfrac{1}{n}}\to 1$$

$$|x-(-2)|<1$$

$$|x+2|<1$$

$$-1<x+2<1$$

$$-3<x<-1$$



$$\text{hax} = -3 \qquad \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\cdot(-3+2)^n = \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$$

- alternál
- $\lim\limits_{n\to\infty}\frac{(-1)^n}{\sqrt{n}} = 0$ mert $\sqrt{n}\to\infty$
- $|a_n| = \frac{1}{\sqrt{n}}$ mon. csökk. mert $\sqrt{n}$ mon. nő

-3-ban Leibniz => konv.

$x = -1 \quad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \cdot (-1+2)^n = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ $= p < 1 \Rightarrow$

$\Rightarrow$ divergens (harmónikus sor)

konv. tartomány $[-3, -1)$