Típus

- geometriai $\sum_{n=0}^{\infty} a_0 \cdot q^n$ ($|q|<1$ konv.)
- Leibniz
  - alternáló
  - $\lim_{n\to\infty} |a_n| = 0$
  - mon. csökk. $|a_n|$

  } konv.

(ha $\lim_{n\to\infty} |a_n| \neq 0$ akkor div.)

- Pozitív tagú :
  - hányados / gyökkrit
  - majoráns / minoráns

11/7 feladat

a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3}}$

Leibniz – alternálás $(-1)^n$ – előjelváltó

$\frac{1}{\sqrt{n^3}} > 0$

$|a_n| = \frac{1}{\sqrt{n^3}}$

– $\lim_{n\to\infty} \frac{\overset{\to 1}{1}}{\underset{\to\infty}{\sqrt{n^3}}} = \frac{1}{\infty} = 0$

$a_1 = -1 \quad a_2 = \frac{1}{2\sqrt{2}} \quad a_3 = \frac{-1}{3\sqrt{3}}$

– $|a_n| > |a_{n+1}|$

$\frac{1}{\sqrt{n^3}} > \frac{1}{\sqrt{(n+1)^3}}$

$\Rightarrow$ Leibniz sor tehát konvergens

$(n+1)^{3/2} > n^{3/2}$

$n+1 > n$ mon. csökken

$$\Rightarrow \sum |a_n| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}} \to \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} = a$$

ha $a > 1$ konv.
ha $a \leq 1$ div.

az abszolút sor konvergens, mert $a > 1$
$\Rightarrow$ az eredeti sor ABSZOLÚT konvergens.

b)
$$\sum_{n=2}^{\infty} \frac{3^n}{n!} = \frac{3^2}{2!} + \frac{3^3}{3!} + \dots = \frac{9}{2} + \frac{27}{6} + \frac{81}{24} + \dots$$

pozitív tagú sor! $\sum a_n = \sum |a_n|$ $\to$ div. / $\to$ absz. konv.

Praktikusan: HÁNYADOS krit.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} = \lim_{n\to\infty} \frac{\overbrace{3 \cdot 3 \cdots 3}^{n+1}}{1 \cdot 2 \cdot 3 \cdots n (n+1)} \cdot \frac{1 \cdot 2 \cdots n}{\underbrace{3 \cdots 3}_{n}} =$$

$$= \lim_{n\to\infty} \frac{3}{n+1} = \frac{3}{\infty} = 0 < 1 \to \text{konv.}$$

($>1 \to$ div.
$=1 \to$ nem tudjuk)

$\Rightarrow$ a sor ABSZOLÚT konvergens.

11/4 (Hf) $\sum_{n=1}^{\infty} \frac{2+3^{n-1}}{5^n}$ igazoljuk, hogy konvergens ÉS sorösszeg kiszámítása!

$$\sum_{n=0}^{\infty} a_0 \cdot q^n = \begin{cases} |q|<1 \text{ konv.} \Rightarrow a_0 \cdot \frac{1}{1-q} \\ |q| \geq 1 \text{ div.} \end{cases}$$

$$\| \\ a_0 + a_0 \cdot q + a_0 \cdot q^2 + \dots$$

$$\sum_{n=1}^{\infty} \frac{2}{5^n} + \frac{3^{n-1}}{5^n}$$

$$\sum_{n=1}^{\infty} \frac{2}{5^n} = \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \dots$$

$a_0 = 2$, $q = \frac{1}{5}$ $(|q|<1)$

$$\left(\sum_{n=0}^{\infty} 2 \cdot \frac{1}{5^n}\right) - 2 = 2 \cdot \frac{1}{1-\frac{1}{5}} - 2 = \cancel{2} \cdot \frac{5}{\cancel{4}2} - 2 = \frac{1}{2}$$

VAGY $\sum_{n=0}^{\infty} \underbrace{\frac{2}{5}}_{a_0} \cdot \frac{1}{5}^n = \frac{2}{5} \cdot \frac{1}{1-\frac{1}{5}} = \frac{2}{5} \cdot \frac{1}{\frac{4}{5}} = \frac{2}{5} \cdot \frac{5}{4} = \frac{1}{2}$ (q)

$$\sum_{n=1}^{\infty} \frac{3^{n-1}}{5^n} = \underbrace{\frac{1}{5}}_{a_0} + \underbrace{\frac{3}{25}}_{=a_0 \cdot \frac{3}{5}} + \underbrace{\frac{9}{125}}_{=a_0 \cdot \left(\frac{3}{5}\right)^2} + \dots$$

$a_0 = \frac{1}{5}$, $q = \frac{3}{5}$ $(|q|<1)$

összeg $\frac{1}{5} \cdot \frac{1}{1-\frac{3}{5}} = \frac{1}{5} \cdot \frac{5}{2} = \frac{1}{2}$

$$\sum_{n=0}^{\infty} \frac{1}{5} \cdot \left(\frac{3}{5}\right)^n$$

Ilyenkor mindkét összegsor konvergens

a végösszeg: $\frac{1}{2} + \frac{1}{2} = 1$

11 fs / 6 d)

$$\sum_{n=1}^{\infty} (-1)^n \cdot \left(\frac{n+1}{n+4}\right)^n = -1 \cdot \left(\frac{2}{5}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{4}{7}\right)^3 + \left(\frac{5}{8}\right)^4 - \dots$$

- alternál

DE $\lim\limits_{n \to \infty} \left(\frac{n+1}{n+4}\right)^n = \lim\limits_{n \to \infty} \frac{\left(\frac{n+1}{n}\right)^n}{\left(\frac{n+4}{n}\right)^n} = \lim\limits_{n \to \infty} \frac{\left(1+\frac{1}{n}\right)^n}{\left(1+\frac{4}{n}\right)^n} = \frac{e}{e^4} = \frac{1}{e^3} \neq 0$

Szükséges feltétel ($\lim\limits_{n \to \infty} a_n = 0$) NEM teljesül

$\Rightarrow$ a sor divergens!

Diagonalizálás

$$\underline{\underline{A}} = \begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix}$$

1) Sajátérték $\quad |\underline{\underline{A}} - \lambda \underline{\underline{E}}| = \begin{vmatrix} 2-\lambda & -3 \\ -3 & 2-\lambda \end{vmatrix} =$

$$(2-\lambda)^2 - 9 = 4 - 4\lambda + \lambda^2 - 9 =$$

$$\lambda^2 - 4\lambda - 5 = (\lambda - 5)(\lambda + 1)$$

$$\underbrace{\text{s.é.}}\quad \lambda_1 = 5 \quad \lambda_2 = -1$$

2) Sajátvektor

$\lambda_1 = 5$

$$(\underline{\underline{A}} - 5\underline{\underline{E}})\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -3 & -3 \\ -3 & -3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \longrightarrow \left.\begin{matrix} -3x - 3y = 0 \\ -3x - 3y = 0 \end{matrix}\right\} \; x = -y$$

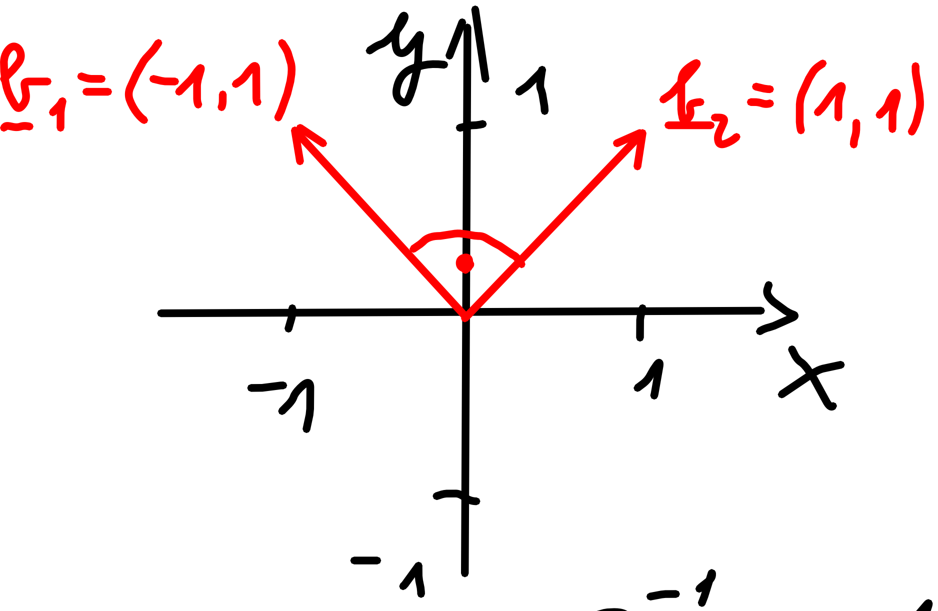
$$\underline{s} = \begin{pmatrix} -y \\ y \end{pmatrix} \quad y \neq 0 \longrightarrow \underline{b}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad \text{Bázisvektor 1.}$$

$\lambda_1 = -1$

$$(\underline{\underline{A}} - (-1)\underline{\underline{E}})\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \longrightarrow \left.\begin{matrix} 3x - 3y = 0 \\ -3x + 3y = 0 \end{matrix}\right\} \; x = y$$

$$\underline{s} = \begin{pmatrix} x \\ x \end{pmatrix} \quad x \neq 0 \longrightarrow \underline{b}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{Bázisvektor 2.}$$



3, Diagonalizálás

$$\underline{\underline{B}} = \left(\underline{b}_1 \,\middle|\, \underline{b}_2\right) = \begin{pmatrix} -1 & 1 \\ \underbrace{1}_{\underline{b}_1} & \underbrace{1}_{\underline{b}_2} \end{pmatrix}$$

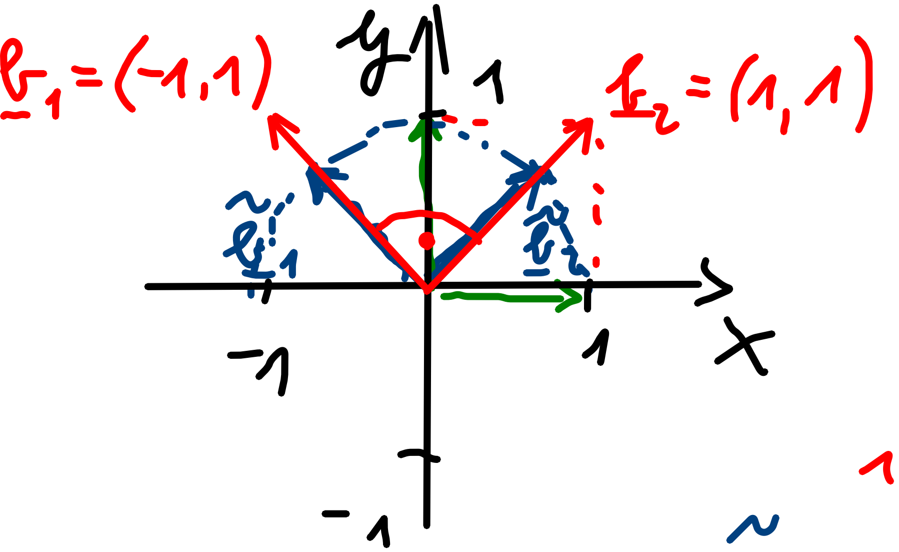
$\underline{\underline{B}}^{-1} \cdot \underline{\underline{A}} \cdot \underline{\underline{B}}$ hagyományosan

$$\underline{\underline{B}}^{-1} = \frac{1}{\det \underline{\underline{B}}} \cdot \operatorname{adj} \underline{\underline{B}} = \frac{1}{-2} \cdot \begin{pmatrix} +1 & -1 \\ -1 & +(-1) \end{pmatrix}^T = -\frac{1}{2} \cdot \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}^T =$$

$$= \left(-\frac{1}{2}\right) \cdot \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} = 1/2 \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\underline{\underline{B}}^{-1} \cdot \underline{\underline{A}} \cdot \underline{\underline{B}} =$$

$$\begin{array}{c|c|c} & \underline{\underline{A}} = \begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix} & \underline{\underline{B}} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \\ \hline \underline{\underline{B}}^{-1} = \begin{pmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} & \begin{pmatrix} -5/2 & 5/2 \\ -1/2 & -1/2 \end{pmatrix} & \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix} \end{array} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$



3, Újra diagonalizálás, de

ORTHONORMÁLT bázissal!

Csak szimmetrikus $\underline{\underline{A}}$-ra $\left(\begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix}\right)$

ilyenkor $\underline{b}_1 \perp \underline{b}_2$ MINDIG

$$|\underline{b}_1| = |(-1,1)| = \sqrt{(-1)^2+(1)^2} = \sqrt{2} \qquad \tilde{\underline{b}}_1 = \frac{\underline{b}_1}{|\underline{b}_1|} = \frac{1}{\sqrt{2}}(-1,1) = \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$|\underline{b}_2| = |(1,1)| = \sqrt{1^2+1^2} = \sqrt{2} \qquad \tilde{\underline{b}}_2 = \frac{\underline{b}_2}{|\underline{b}_2|} = \frac{1}{\sqrt{2}}(1,1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

ortho normált: $\tilde{\underline{b}}_1 \perp \tilde{\underline{b}}_2$, $|\tilde{\underline{b}}_1| = |\tilde{\underline{b}}_2| = 1$

$$\tilde{\underline{\underline{B}}} = (\tilde{\underline{b}}_1, \tilde{\underline{b}}_2) = \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \Rightarrow \tilde{\underline{\underline{B}}}^{-1} = \tilde{\underline{\underline{B}}}^{T} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\tilde{\underline{\underline{B}}}^{-1} \underline{\underline{A}} \tilde{\underline{\underline{B}}} = \tilde{\underline{\underline{B}}}^{T} \cdot \underline{\underline{A}} \cdot \tilde{\underline{\underline{B}}} =$$

$$\begin{array}{c|c|c} & \underline{\underline{A}} = \begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix} & \tilde{\underline{\underline{B}}} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \\ \hline \tilde{\underline{\underline{B}}}^{T} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} & \begin{matrix} -5 & 5 \\ -1 & -1 \end{matrix} & \begin{pmatrix} 10 & 0 \\ 0 & -2 \end{pmatrix} \cdot \underbrace{\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}}_{1/2} \end{array} = \frac{1}{2} \begin{pmatrix} 10 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}$$