- geometriai sorok $\sum a_0 \cdot q^n$

$|q|<1$ ↳ konvergens

$|q|\geq 1$ ↳ divergens

$$\sum_{n=1}^{\infty} \frac{2+3^n}{5^{n+1}} = \sum_{n=1}^{\infty} \frac{2}{5\cdot 5^n} + \frac{3^n}{5\cdot 5^n} =$$

$$= \sum_{n=1}^{\infty} \underbrace{\frac{2}{5}\cdot\left(\frac{1}{5}\right)^n}_{\text{konv}} + \underbrace{\frac{1}{5}\left(\frac{3}{5}\right)^n}_{\text{konv}} = \frac{1}{10} + \frac{3}{10} = \frac{4}{10} = \frac{2}{5}$$

$$\sum_{n=1}^{\infty} \frac{2}{5}\cdot\left(\frac{1}{5}\right)^n = \frac{2}{5}\cdot\frac{1}{5} + \frac{2}{5}\cdot\left(\frac{1}{5}\right)^2 \cdots\cdots = \underbrace{\sum_{n=0}^{\infty} \frac{2}{25}\cdot\left(\frac{1}{5}\right)^n}$$

$q = \frac{1}{5}$ $\quad = a_0 = \frac{2}{25}$

$$a_0\cdot\frac{1}{1-q} = \frac{2}{25}\cdot\frac{1}{1-\frac{1}{5}} = \frac{\cancel{2}}{\cancel{25}}\cdot\frac{\cancel{5}}{\cancel{4}_2} = \frac{1}{10}$$

$$\sum_{n=1}^{\infty} \frac{1}{5}\cdot\left(\frac{3}{5}\right)^n = \frac{1}{5}\cdot\frac{3}{5} + \frac{1}{5}\cdot\frac{3^2}{5^2} + \cdots = \frac{3}{25}\cdot\frac{1}{1-\frac{3}{5}} = \frac{3}{\cancel{25}}\cdot\frac{\cancel{5}}{2} = \frac{3}{10}$$

$\frac{3}{5} = q$ $\quad \frac{1}{5}\cdot\frac{3}{5} = \frac{3}{25} = a_0$

• pozitív tagú - e ?

igen

• $\sum_{n=1}^{\infty} \frac{1}{2n+1} = \frac{1}{3}+\frac{1}{5}+\frac{1}{7}\dots$

• Hányados - vagy Gyökkritériummal

- $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = H$

Ha $H<1$ $\Rightarrow$ konv.

Ha $H>1$ $\rightarrow$ div.

Ha $H=1$???

- $\lim_{n\to\infty} \sqrt[n]{a_n} = H$

• Minoráns és Majoráns krit

$\sum_{n=1}^{\infty} \frac{1}{n^a}$ ha "a" >1 $\rightarrow$ konv.

ha "a" ≤ 1 $\rightarrow$ div.

abszolút konv ?

$\sum_{n=1}^{\infty} |a_n|$

• $\sum a_n$ és $\sum |a_n|$ is konv = ABSZ. konv.

• $\sum a_n$ konv. DE $\sum |a_n|$ div = FELT. konv.

• $\sum a_n$ div $\Rightarrow$ $\sum |a_n|$ is div = DIV

nem

• $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n}{2n+1} = -\frac{1}{3}+\frac{2}{5}-\frac{3}{7}+\frac{4}{9}\dots$ DIV.

• Leibniz-sor - e ? $\nearrow$ igen $\rightarrow$ konvergens

$\searrow$ nem ???

- alternáló • $(-1)^n$ miatt OK

- $\lim_{n\to\infty} |a_n| = 0$ • $\lim_{n\to\infty} \frac{n}{2n+1} = \frac{\infty}{\infty} = \lim_{n\to\infty} \frac{1}{2+\frac{1}{n}} = \frac{1}{2}$ ($\frac{1}{n} \to 0$)

$\hookrightarrow$ nem teljesül $\hookrightarrow$ divergens

- $|a_n|$ mon. csökkenő sorozat

mindhárom OK $\hookrightarrow$ konvergens

• $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{2n+1} = -\frac{1}{3}+\frac{1}{5}-\frac{1}{7}\dots$

• alternál $(-1)^n$ OK

• $\lim_{n\to\infty} \frac{1}{2n+1} = \frac{1}{\infty} = 0$ ok

• mon. csökk. ok

$|a_n| = \frac{1}{2n+1} \overset{?}{>} |a_{n+1}| = \frac{1}{2n+3}$ akkor mert $2n+1 < 2n+3$

KONV.

Ha $b_n > a_n$ $\forall n$-re és $\sum b_n$ konv $\rightarrow$ $\sum a_n$ is konv. MAJORÁNS

Ha $a_n > b_n$ $\forall n$-re és $\sum b_n$ div $\rightarrow$ $\sum a_n$ is div MINORÁNS

$\sum_{n=1}^{\infty} \frac{1}{2n+1} = ?$ Mit mond a ...

- Hányados krit: $\lim_{n\to\infty} \dfrac{\frac{1}{2n+3}}{\frac{1}{2n+1}} = \lim_{n\to\infty} \frac{1}{2n+3} \cdot \frac{2n+1}{1} = \lim_{n\to\infty} \frac{2n+1}{2n+3} = \frac{\infty}{\infty} =$

$$= \lim_{n\to\infty} \frac{2+\frac{1}{n}}{2+\frac{3}{n}} = 1 \Rightarrow \text{semmit!}$$

- Gyökkrit: $\lim_{n\to\infty} \sqrt[n]{\frac{1}{2n+1}} = \lim_{n\to\infty} \frac{1}{\sqrt[n]{2n+1}} = 1 \Rightarrow$ semmit!

$$\lim_{n\to\infty} \sqrt[n]{a} = 1 \qquad \underset{\downarrow \atop 1}{\sqrt[n]{n}} < \underset{n\to\infty \atop {\downarrow \atop 1}}{\sqrt[n]{2n+1}} < \sqrt[n]{3n} = \underbrace{\underset{\downarrow \atop 1}{\sqrt[n]{3}} \cdot \underset{\downarrow \atop 1}{\sqrt[n]{n}}}_{\to 1}$$

- Min. v. majoráns $\overset{?}{<} \underbrace{\frac{1}{2n+1}}_{} \overset{?}{<}$

$$\sim \frac{n^0}{n^1} = \frac{1}{n} \qquad \sum \frac{1}{n} \text{ div}$$

$$\underline{\underline{\frac{1}{3n}}} = \frac{1}{2n+n} < \frac{1}{2n+1} \qquad \sum_{n=1}^{\infty} \frac{1}{3n} = \sum_{n=1}^{\infty} \frac{1}{n} \cdot \frac{1}{3} =$$

$$= \frac{1}{3} \cdot \sum_{n=1}^{\infty} \frac{1}{n} = \infty \text{ div.}$$

ezért $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{2n+1}$ is divergens

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{(n+1)!} = \frac{-2}{2} + \frac{4}{6} - \frac{8}{24} + \frac{16}{120} - \dots$$ nem geometriai poz. tagú? NEM

$$= -1 + \frac{2}{3} - \frac{1}{3} + \frac{2}{15} - \dots$$

- Leibniz-e IGEN $\rightarrow$ konvergens

  - alternál: $(-2)^n = (-1\cdot 2)^n = \underbrace{(-1)^n}_{\text{alt.}} \cdot \underbrace{2^n}_{>0}$ ✓OK

$|a_n| = \frac{2^n}{(n+1)!}$ – $\lim\limits_{n\to\infty} \frac{2^n}{(n+1)!} = \lim\limits_{n\to\infty} \frac{2\cdot 2 \cdots 2}{1\cdot 2 \cdots n\cdot(n+1)} = \frac{\infty}{\infty} =$

$$= \lim_{n\to\infty} \frac{1}{\frac{1}{2}\cdot 1\cdot\frac{3}{2}\cdot\frac{4}{2}\cdots\frac{n}{2}(n+1)} = \frac{1}{\infty} = 0 \quad \text{OK}$$

  - mon. csökk. $|a_n| \geq |a_{n+1}|$

$$\frac{2^n}{(n+1)!} \geq \frac{2^{n+1}}{(n+2)!} \quad /:2^n$$

$$(n+2)! \cdot 1 \geq 2\cdot(n+1)!$$

$$\cancel{1}\cdot\cancel{2}\cdots\cancel{(n+1)}(n+2) \geq 2\cdot\cancel{1}\cdot\cancel{2}\cdots\cancel{(n+1)}$$

$$n+2 \geq 2 \quad ✓$$

OK

$$\sum_{n=1}^{\infty} \frac{2^n}{(n+1)!} = \frac{2}{2} + \frac{4}{6} + \frac{6}{24} + \frac{8}{120} + \dots$$

$\downarrow$ fakt. miatt hányados krit.

$$\lim_{n \to \infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n \to \infty} \frac{\frac{2^{n+1}}{(n+2)!}}{\frac{2^n}{(n+1)!}} = \lim_{n \to \infty} \frac{2^{\cancel{n+1}}}{\underbrace{(n+2)!}_{\cancel{1}\cdot\ldots\cdot(\cancel{n+1})(n+2)}} \cdot \frac{\overbrace{(n+1)!}^{\cancel{1}\cdot\ldots\cdot(\cancel{n+1})}}{\cancel{2^n}} =$$

$$= \lim_{n \to \infty} \frac{2}{n+2} = \frac{2}{\infty} = 0 < 1 \Rightarrow \text{ezért } \sum_{n=1}^{\infty} |a_n| \text{ is}$$

konvergens

$\Rightarrow$ az eredeti sor abszolút konvergens.