$$\underline{\underline{A}}\cdot\underline{x} = \underline{b} + \underline{x} \qquad \underline{\underline{A}} = \begin{pmatrix} 3 & 1 & 1 \\ -2 & 4 & -4 \\ 1 & 1 & 1 \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}$$

$$\underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\underline{\underline{A}}\cdot\underline{x} = \begin{pmatrix} 3 & 1 & 1 \\ -2 & 4 & -4 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3x+y+z \\ -2x+4y-4z \\ x+y+z \end{pmatrix} \qquad \underline{b}+\underline{x} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x-2 \\ y \\ z-1 \end{pmatrix}$$

$$\begin{aligned} 3x+y+z &= x-2 \\ -2x+4y-4z &= y \\ x+y+z &= z-1 \end{aligned} \quad \longrightarrow \quad \begin{aligned} 2x+y+z &= -2 \\ -2x+3y-4z &= 0 \\ x+y &= -1 \end{aligned}$$

$$\underline{\underline{A}}\,\underline{x} = \underline{b} + \underline{x} \qquad / -\underline{x}$$

$$\underbrace{\underline{\underline{A}}\cdot\underline{x} - \underline{x}}_{} = \underline{b}$$

$$\underline{\underline{A}}\,\underline{x} - \underbrace{\underline{\underline{E}}}_{\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}}\underline{x} = \left(\underline{\underline{A}} - \underline{\underline{E}}\right)\underline{x} = \left(\begin{pmatrix}3&1&1\\-2&4&-4\\1&1&1\end{pmatrix} - \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right)\underline{x} =$$

$$= \underbrace{\begin{pmatrix}2&1&1\\-2&3&-4\\1&1&0\end{pmatrix}}_{\underline{\underline{C}}}\underline{x} = \underline{b} = \begin{pmatrix}-2\\0\\-1\end{pmatrix}$$

$$\underline{\underline{C}}^{-1}\underline{\underline{C}}\,\underline{x} = \underline{\underline{C}}^{-1}\underline{b}$$

$$\underline{\underline{E}}\,\underline{x} = \underline{\underline{C}}^{-1}\cdot\underline{b}$$

$$\underline{x} = \underline{\underline{C}}^{-1}\cdot\underline{b}$$

Sor : $\{a_n\} = \{a_1, a_2, \dots\} \to \sum_{n=1}^{\infty} a_n$

sorozat

$s_1 = a_1$

$s_2 = a_1 + a_2$

$s_3 = a_1 + a_2 + a_3$

$\vdots$

részlet-összeg $\dashrightarrow s_n \to \{s_n\}$ hova tart

Geometriai sor:

$$\sum_{n=0}^{\infty} A \cdot q^n = A + A \cdot q + A \cdot q^2 + \dots$$

$$\sum_{n=0}^{\infty} A \cdot \left(-\frac{1}{2}\right)^n \to \text{konvergens, ha } |q| < 1$$

$$\sum_{n=0}^{\infty} \frac{456}{7} \cdot \left(-\frac{6}{13}\right)^n \to \text{konvergens!}$$

$$\sum_{n=0}^{\infty} A \cdot q^n = A \cdot \frac{1}{1-q} \quad \text{összeg}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}} = \begin{cases} \text{div. ha } \alpha \leq 1 \\ \text{konv. ha } \alpha > 1 \end{cases}$$

$$\sum \frac{1}{\sqrt[6]{n^5}} = \frac{1}{1} + \frac{1}{\sqrt[6]{2^5}} + \frac{1}{\sqrt[6]{3^5}} \dots$$

divergens

Stratégiák

$$\sum_{n=1}^{\infty} \underbrace{\frac{(-1)^n}{2^n+1}}_{a_n} \xrightarrow[\text{sor}]{\text{abszolút}} \sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \left| \frac{(-1)^n}{2^n+1} \right| = \sum_{n=1}^{\infty} \frac{1}{2^n+1}$$

Ha $\sum a_n$ konv de $\sum |a_n|$ divergens

$\Rightarrow \sum a_n$ feltételesen konv.

* Ha $\sum a_n$ konv $\cancel{\Leftarrow}$ $\sum |a_n|$ konv.

$\Rightarrow \sum a_n$ abszolút konv.

Ha $\sum a_n$ div $\rightarrow \sum |a_n|$ is az

$\Rightarrow \sum a_n$ divergens

Leibniz-sor ha: – alternál ps < 0 és ptl > 0 vagy fordítva

Ha van ∞ sok negatív tag!

kell!

Példában $\frac{(-1)^n}{2^n+1}$ ← alternál } ✓ ok

$2^n+1>0$

$a_1=\frac{-1}{3}$ $a_2=\frac{1}{5}$ $a_3=\frac{-1}{9}$

– $\lim\limits_{n\to\infty}|a_n|=0$ szükséges!

Példa $\lim\limits_{n\to\infty}\frac{1}{2^n+1}=\frac{1}{\infty}=0$ ✓ OK

– $|a_n|$ szig. mon. csökk. sorozat

Példa $|a_n|>|a_{n+1}|$

$\frac{1}{2^n+1}>\frac{1}{2^{n+1}+1}$ } ✓ OK

$2^n+1<2^{n+1}+1$ mon. nő a nevező

$$\sum |a_n| = \sum_{n=1}^{\infty} \frac{1}{2^n+1} = \frac{1}{3} + \frac{1}{5} + \frac{1}{9} \cdots$$ pozitív tagú sor

Hányados / gyökkritérium

- Hányados-krit: $\lim\limits_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = H \leftarrow \begin{cases} \text{ha } H<1 \to \text{konv.} \\ \text{ha } H>1 \to \text{div.} \\ \text{ha } H=1 \to \text{nem tudjuk} \end{cases}$

$$\lim_{n\to\infty} \frac{\frac{1}{2^{n+1}+1}}{\frac{1}{2^n+1}} = \lim_{n\to\infty} \frac{2^n+1}{\underbrace{2^{n+1}}_{2\cdot 2^n}+1} = \frac{\infty}{\infty} = \lim_{n\to\infty} \frac{1+\frac{1}{2^n}}{2+\frac{1}{2^{n}}} = \frac{1+0}{2+0} = \frac{1}{2} < 1$$

$$\Rightarrow \sum |a_n| \text{ konv.}$$

- Gyök-krit: $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} = H$

$$\lim_{n\to\infty} \sqrt[n]{\frac{1}{2^n+1}} = \lim_{n\to\infty} \frac{1}{\sqrt[n]{2^n+1}} = \frac{1}{2}$$

$$\sqrt[n]{2^n} = 2$$

$$\underset{\downarrow \atop 2}{\sqrt[n]{2^n}} \le \sqrt[n]{2^n+1} \le \sqrt[n]{2^n+2^n} = \sqrt[n]{2\cdot 2^n} = \underset{\downarrow \atop 2\,\cdot\,1}{2\cdot\sqrt[n]{2}}$$

Minoráns / Majoráns

Majoráns

$0 < \sum |a_n|$ Ha van $\sum b_n$ amire

pozitív tagú

$|a_n| < b_n$ és $\underbrace{\sum b_n \text{ konv.}}_{\text{majoráló}}$

$\Rightarrow \sum |a_n|$ is konvergens

$\sum \frac{1}{2^n+1}$ $\quad \frac{1}{2^n+1} < \frac{1}{2^n} = \left(\frac{1}{2}\right)^n \quad \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$

Geometriai $q = \frac{1}{2} < 1$

tehát konvergens

Minoráns

$\sum |a_n|$-re létezik $\sum b_n$, hogy $|a_n| > b_n > 0$

és $\sum b_n$ divergens $\Rightarrow \sum |a_n|$ is divergens

$\sum \frac{1}{n+1}$ $\quad \frac{1}{n+1} > \frac{1}{n+n} = \frac{1}{2n} \quad \sum_{n=1}^{\infty} \frac{1}{2} \cdot \frac{1}{n}$ divergens

mert $\sum \frac{1}{n}$ is az