Matematika A2 Vizsga GTK NG&PSz MEGOLDÁSOK 2024.05.28.

1.feladat

$$\int_{-2}^{2} \frac{3}{\sqrt{2-x}}\,dx = \lim_{\varepsilon \to 0+} \int_{-2}^{2-\varepsilon} \frac{3}{\sqrt{2-x}}\,dx = \lim_{\varepsilon \to 0+} \left[-\frac{3\sqrt{2-x}}{\frac{1}{2}} \right]_{-2}^{2-\varepsilon} =$$

$$= \lim_{\varepsilon \to 0+} -6\sqrt{2-(2-\varepsilon)} + 6\sqrt{2-(-2)} = \lim_{\varepsilon \to 0+} -6\underbrace{\sqrt{\varepsilon}}_{\to 0} + 6\sqrt{4} = 12$$

2.feladat

$$\left.\begin{array}{l} P(3,4,1) \\ \underline{n}(2,-1,2) \end{array}\right\} \quad \alpha \text{ sík: } \begin{array}{l} 2(x-3)-(y-4)+2(z-1)=0 \\ 2x-y+2z-4=0 \end{array}$$

$Q(-1,3,-5)$ normálalak: $|\underline{n}| = \sqrt{2^2+(-1)^2+2^2} = \sqrt{9} = 3$

$$d(\alpha, Q) = \left| \frac{2\cdot(-1)-3+2\cdot(-5)-4}{3} \right| = \left| \frac{-2-3-10-4}{3} \right| = \frac{19}{3}$$

$\beta \parallel \alpha$ sík: $2(x+1)-(y-3)+2(z+5)=0$

$Q \in \beta$ $\qquad 2x-y+2z+15=0$

3.feladat

$$\det(\underline{\underline{A}}) = \begin{vmatrix} 4 & -2 & 1 \\ -2 & 0 & -3 \\ 3 & p & 4 \end{vmatrix} = -(-2)\begin{vmatrix} -2 & 1 \\ p & 4 \end{vmatrix} - (-3)\begin{vmatrix} 4 & -2 \\ 3 & p \end{vmatrix} = 2(-8-p)+3(4p+6) =$$

$$= -16 - 2p + 12p + 18 = 10p + 2 \qquad p = \frac{-2}{10} = \frac{-1}{5}\text{-re}$$

szinguláris a mátrix

ha $p=-1$

$$\underline{\underline{A}} = \begin{pmatrix} 4 & -2 & 1 \\ -2 & 0 & -3 \\ 3 & -1 & 4 \end{pmatrix} \qquad \det(\underline{\underline{A}}) = 10\cdot(-1)+2 = -8$$

$$\underline{\underline{A}}^{-1} = \frac{1}{-8}\cdot\begin{pmatrix} \begin{vmatrix} 0 & -3 \\ -1 & 4 \end{vmatrix} & -\begin{vmatrix} -2 & -3 \\ 3 & 4 \end{vmatrix} & \begin{vmatrix} -2 & 0 \\ 3 & -1 \end{vmatrix} \\ -\begin{vmatrix} -2 & 1 \\ -1 & 4 \end{vmatrix} & \begin{vmatrix} 4 & 1 \\ 3 & 4 \end{vmatrix} & -\begin{vmatrix} 4 & -2 \\ 3 & -1 \end{vmatrix} \\ \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} & -\begin{vmatrix} 4 & 1 \\ -2 & -3 \end{vmatrix} & \begin{vmatrix} 4 & -2 \\ -2 & 0 \end{vmatrix} \end{pmatrix}^{T} = -\frac{1}{8}\cdot\begin{pmatrix} -3 & -1 & 2 \\ 7 & 13 & -2 \\ 6 & 10 & -4 \end{pmatrix}^{T} = -\frac{1}{8}\cdot\begin{pmatrix} -3 & 7 & 6 \\ -1 & 13 & 10 \\ 2 & -2 & -4 \end{pmatrix} =$$

$$= \begin{pmatrix} 3/8 & -7/8 & -3/4 \\ 1/8 & -13/8 & -5/4 \\ -1/4 & 1/4 & 1/2 \end{pmatrix} \qquad \text{lesz az inverz } p=-1\text{-re.}$$

4.feladat

$$\det(\underline{\underline{A}} - \lambda \underline{\underline{E}}) = \begin{vmatrix} -1-\lambda & 3 & 0 \\ 3 & -5-\lambda & 2 \\ 0 & 2 & 1-\lambda \end{vmatrix} = (-1-\lambda)\cdot \begin{vmatrix} -5-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} - 3 \cdot \begin{vmatrix} 3 & 2 \\ 0 & 1-\lambda \end{vmatrix} =$$

$$= (-1-\lambda)\left[\lambda^2 + 4\lambda - 5 - 4\right] - 3\cdot 3(1-\lambda) = -\lambda^3 - \underline{4\lambda^2} + \underline{9\lambda} - \underline{\lambda^2} - \underline{4\lambda} + \cancel{9} - \cancel{9} + \underline{9\lambda} =$$

$$= -\lambda^3 - 5\lambda^2 + 14\lambda = -\lambda\cdot(\lambda^2 + 5\lambda - 14) = -\lambda(\lambda+7)(\lambda-2)$$

$$\lambda_1 = -7\,;\ \lambda_2 = 0\,;\ \lambda_3 = 2$$

Ha $\lambda_1 = -7$

$$\begin{pmatrix} 6 & 3 & 0 \\ 3 & 2 & 2 \\ 0 & 2 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left.\begin{aligned} 6x + 3y &= 0 \rightarrow x = -\tfrac{1}{2}y \\ 3x + 2y + 2z &= 0 \\ 2y + 8z &= 0 \rightarrow z = -\tfrac{1}{4}y \end{aligned}\right\}$$

$$3\left(-\tfrac{1}{2}y\right) + 2y + 2\left(-\tfrac{1}{4}y\right) = 0 \checkmark$$

$$\underline{s}_{-7} = \begin{pmatrix} -1/2\, y \\ y \\ -1/4\, y \end{pmatrix},\ y \in \mathbb{R}\setminus\{0\}$$

Ha $\lambda_2 = 0$

$$\begin{pmatrix} -1 & 3 & 0 \\ 3 & -5 & 2 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left.\begin{aligned} -x + 3y &= 0 \rightarrow x = 3y \\ 3x - 5y + 2z &= 0 \\ 2y + z &= 0 \rightarrow z = -2y \end{aligned}\right\}$$

$$\overset{9}{3\cdot(3y)} - 5y + \overset{-4}{2(-2y)} = 0 \checkmark$$

$$\underline{s}_0 = \begin{pmatrix} 3y \\ y \\ -2y \end{pmatrix},\ y \in \mathbb{R}\setminus\{0\}$$

Ha $\lambda_3 = 2$

$$\begin{pmatrix} -3 & 3 & 0 \\ 3 & -7 & 2 \\ 0 & 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left.\begin{aligned} -3x + 3y &= 0 \rightarrow x = y \\ 3x - 7y + 2z &= 0 \\ 2y - z &= 0 \rightarrow z = 2y \end{aligned}\right\}$$

$$3\cdot y - 7y + \overset{4}{2(2y)} = 0 \checkmark$$

$$\underline{s}_2 = \begin{pmatrix} y \\ y \\ 2y \end{pmatrix},\ y \in \mathbb{R}\setminus\{0\}$$

5.feladat

$f(x,y) = x^2y - y^2 - 8xy$

$f'_x = 2xy - 8y$

$f'_y = x^2 + 2y - 8x$

$\left.\begin{matrix} 2y(x-4) = 0 \\ x^2 - 8x + 2y = 0 \end{matrix}\right\}$ $\nearrow y = 0$, $\searrow x = 4$

akkor $x^2 - 8x = 0$

$x(x-8) = 0$

$x_1 = 0 \quad x_2 = 8$

$P_1(0,0)$; $P_2(8,0)$

akkor $16 - 32 + 2y = 0$

$y = 8$

$P_3(4, 8)$

stac. pontok

$f''_{xx} = 2y$

$f''_{xy} = 2x - 8 = f''_{yx}$

$f''_{yy} = 2$

• P_1-re $\det(\text{Hesse}(0,0)) = \begin{vmatrix} 0 & -8 \\ -8 & 2 \end{vmatrix} = -64 < 0$ nyeregpont

• P_2-re $\det(\text{Hesse}(8,0)) = \begin{vmatrix} 0 & 8 \\ 8 & 2 \end{vmatrix} = -64 < 0$ nyeregpont

• P_3-ra $\det(\text{Hesse}(4,8)) = \begin{vmatrix} 16 & 0 \\ 0 & 2 \end{vmatrix} = 32 > 0$ és $f''_{xx} > 0$

lok. min. hely

$f(4,8) = 128 + 64 - 256 = -64$ lok. min. értéke

6.feladat

$\sum_{n=1}^{\infty} \frac{n}{3^n} \cdot (x-5)^n \quad a_n = \frac{n}{3^n}$; $x_0 = 5$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \underbrace{\frac{n}{n+1}}_{\to 1} \cdot \underbrace{\frac{3^n}{3^n+1}}_{\to \frac{1}{3}} = \frac{1}{3}$

Konvergenciasugár: $\frac{1}{1/3} = 3$

$|x-5| < 3$

$x = 2 \quad \sum_{n=1}^{\infty} \frac{n}{3^n} \cdot (-3)^n = \sum_{n=1}^{\infty} (-1)^n n$ divergens (ált. tag $\not\to 0$)

$x = 8 \quad \sum_{n=1}^{\infty} \frac{n}{3^n} \cdot 3^n = \sum_{n=1}^{\infty} n$ divergens (ált. tag $\not\to 0$)

$\boxed{x \in (3, 8) \text{ konv. tart.}}$