Matematika A2 Vizsga GTK NG&PSz MEGOLDÁSOK 2024.06.04.

## 1.feladat

$$\int_1^{\infty} x\cdot e^{-x}\,dx = \lim_{b\to\infty} \int_1^{b} x\cdot e^{-x} = \lim_{b\to\infty} \left[-x\cdot e^{-x} - e^{-x}\right]_1^{b} = \lim_{b\to\infty} \underbrace{-b\cdot e^{-b} - e^{-b}}_{\to 0} - \left(-e^{-1} - e^{-1}\right) =$$

$$= \frac{2}{e} \text{ konvergens.}$$

$$\int x\cdot e^{-x}\,dx = -x\cdot e^{-x} + \int e^{-x}\,dx = -x\cdot e^{-x} - e^{-x} + c$$

## 2.feladat

$$\underline{n} = \underline{v}_1 \times \underline{v}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 2 & -3 \\ 1 & -1 & 0 \end{vmatrix} = (-3,-3,-4) \qquad P(-2,4,1)$$

$$\left.\begin{array}{l} \alpha \text{ sík } P \in \alpha \\ \underline{n} \perp \alpha \end{array}\right\} \quad -3(x+2) - 3(y-4) - 4(z-1) = 0$$

$$-3x - 3y - 4z + 10 = 0$$

$$3x + 3y + 4z = 10$$

$Q(1,5,1)$

$$d(\alpha, Q) = \left|\frac{(-3)\cdot 1 + (-3)\cdot 5 + (-4)\cdot 1 + 10}{\sqrt{3^2+3^2+4^2}}\right| = \left|\frac{-12}{\sqrt{34}}\right| = \frac{12}{\sqrt{34}}$$

3.feladat

$$\left(\begin{array}{cccc|c} 3 & -1 & 3 & -1 & 1 \\ -2 & -2 & 0 & 3 & -1 \\ 1 & -2 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & 1 & 1 & 0 \\ -2 & -2 & 0 & 3 & -1 \\ 3 & -1 & 3 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & 1 & 1 & 0 \\ 0 & -6 & 2 & 5 & -1 \\ 0 & 5 & 0 & -4 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & 1 & 1 & 0 \\ 0 & -1 & 2 & 1 & 0 \\ 0 & 5 & 0 & -4 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & -2 & 1 & 1 & 0 \\ 0 & -1 & 2 & 1 & 0 \\ 0 & 0 & 10 & 1 & 1 \end{array}\right)$$

$\text{rang}(\underline{\underline{A}}) = \text{rang}(\underline{\underline{A}}|\underline{b}) = 3 < 4$ : vált. száma

$\infty$ sok megoldás, 1 szabad változó

$$10z + u = 1 \qquad -y + 2z + u = 0 \qquad x - 2y + z + u = 0$$

$$u = 1 - 10z \qquad y = 2z + u = 2z + 1 - 10z \qquad x = 2y - z - u$$

$$z \in \mathbb{R} \qquad y = 1 - 8z \qquad x = 2 - 16z - z - 1 + 10z$$

$$x = 1 - 7z$$

4.feladat

$$\det\begin{pmatrix} -1-\lambda & -2 \\ -2 & 2-\lambda \end{pmatrix} = (-1-\lambda)(2-\lambda) - 4 = \lambda^2 - \lambda - 6 = (\lambda+2)(\lambda-3)$$

$$\lambda_1 = -2; \quad \lambda_2 = 3$$

$\lambda_1 = -2$

$$\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \left.\begin{array}{l} x - 2y = 0 \\ -2x + 4y = 0 \end{array}\right\} \quad x = 2y \quad \underline{s} = \begin{pmatrix} 2y \\ y \end{pmatrix} = y\begin{pmatrix} 2 \\ 1 \end{pmatrix}; \; y \in \mathbb{R}\setminus\{0\}$$

$\lambda_2 = 3$

$$\begin{pmatrix} -4 & -2 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \left.\begin{array}{l} -4x - 2y = 0 \\ -2x - y = 0 \end{array}\right\} \quad y = -2x \quad \underline{s} = \begin{pmatrix} x \\ -2x \end{pmatrix} = x \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}; \; x \in \mathbb{R}\setminus\{0\}$$

$$\underline{\underline{B}} = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \qquad \underline{\underline{B}}^{-1} = -\frac{1}{5} \cdot \begin{pmatrix} -2 & -1 \\ -1 & 2 \end{pmatrix}^T = \begin{pmatrix} 2/5 & 1/5 \\ 1/5 & -2/5 \end{pmatrix}$$

$$\underline{\underline{B}}^{-1}\underline{\underline{A}}\underline{\underline{B}} = \begin{pmatrix} -1 & -2 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 2/5 & 1/5 \\ 1/5 & -2/5 \end{pmatrix} \begin{pmatrix} -4/5 & -2/5 \\ 3/5 & -6/5 \end{pmatrix} \begin{pmatrix} -10/5 & 0 \\ 0 & 15/5 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix} \checkmark$$

5.feladat

$$f(x,y) = x \cdot \ln(x+2y) - 1 \qquad P(-1,1) \qquad f(-1,1) = -1 \ln(1) - 1 = -1$$

$$f'_x = \ln(x+2y) + x \cdot \frac{1}{x+2y} \qquad f'_x(-1,1) = \ln(1) + \frac{-1}{1} = -1$$

$$f'_y = x \cdot \frac{1}{x+2y} \cdot 2 \qquad f'_y(-1,1) = \frac{-2}{1} = -2$$

$$\underline{\text{grad}}\, f(-1,1) = (-1,-2)$$

érintősík $\quad z = -1(x+1) - 2(y-1) + (-1)$

$$x + 2y + z = 0$$

$$f'_{\underline{v}}(-1,1) = \left\langle (-1,-2);\ \left(\frac{3}{5}, \frac{-4}{5}\right) \right\rangle = -\frac{3}{5} + \frac{8}{5} = \frac{5}{5} = 1$$

$$|\underline{v}| = \sqrt{9+16} = 5$$

6.feladat

$$\sum_{n=0}^{\infty} \frac{2^{2n+2}+3^{n-1}}{5^{n+1}} = \frac{4}{5}\cdot\sum_{n=0}^{\infty}\left(\frac{4}{5}\right)^n + \frac{1}{3\cdot 5}\sum_{n=0}^{\infty}\left(\frac{3}{5}\right)^n = *$$

Részletösszeg: $s_k = \sum_{n=0}^{k} \frac{2^{2n+2}+3^{n-1}}{5^{n+1}} = \sum_{n=0}^{k} \frac{4}{5}\cdot\left(\frac{4}{5}\right)^n + \sum_{n=0}^{k} \frac{1}{15}\cdot\left(\frac{3}{5}\right)^n$

$n = k$-ig

$$s_k = \sum_{n=0}^{k} a\cdot q^n = a\cdot\frac{1-q^{k+1}}{1-q} = \frac{4}{5}\cdot\frac{1-(4/5)^{k+1}}{1/5} + \frac{1}{15}\cdot\frac{1-(3/5)^{k+1}}{2/5}$$

error

$$\sum_{n=0}^{\infty} a\cdot q^n = a\cdot\frac{1}{1-q}$$

! ha $|q|<1$

$$* = \frac{4}{5}\cdot\frac{1}{1-\frac{4}{5}} + \frac{1}{15}\cdot\frac{1}{1-\frac{3}{5}} = \frac{4}{5}\cdot\frac{5}{1} + \frac{1}{\cancel{15}_{3}}\cdot\frac{\cancel{5}}{2} = \frac{25}{6}$$

$\left|\frac{4}{5}\right|<1$ és $\left|\frac{3}{5}\right|<1$ $\Rightarrow$ a sor konvergens