Matematika A2 Vizsga GTK NG&PSz MEGOLDÁSOK 2024.06.11.

1.feladat

$z = \frac{1}{2} - \frac{1}{2} i \qquad |z| = \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{1}{\sqrt{2}}$

$\sin\varphi = \frac{-1/2}{1/\sqrt{2}} = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}} \rightarrow \varphi = 225^\circ$ vagy 315°

$\varphi = 315^\circ$

$\cos\varphi = \frac{1/2}{1/\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2} \rightarrow \varphi = 45^\circ$ vagy 315°

$z = \frac{1}{\sqrt{2}} \cdot (\cos 315^\circ + i \sin 315^\circ)$

$z^3 = \frac{1}{2\sqrt{2}} \cdot (\cos 945^\circ + i \sin 945^\circ) = \frac{1}{2\sqrt{2}} \cdot (\cos 225^\circ + i \cdot \sin 225^\circ)$

$$\sqrt[3]{z} = \frac{1}{\sqrt[6]{2}} \cdot \left(\cos(105^\circ + k120^\circ) + i \cdot \sin(105^\circ + 120^\circ) \right) = \begin{cases} \frac{1}{\sqrt[6]{2}} \cdot (\cos 105^\circ + i \sin 105^\circ) \\ \frac{1}{\sqrt[6]{2}} \cdot (\cos 225^\circ + i \sin 225^\circ) \\ \frac{1}{\sqrt[6]{2}} \cdot (\cos 345^\circ + i \sin 345^\circ) \end{cases}$$

2.feladat

$\vec{AB} = (1, 3, -4) - (1, 2, -3) = (0, 1, -1)$

$\vec{AC} = (4, 2, 0) - (1, 2, -3) = (3, 0, 3)$

$T_\Delta = \frac{|\vec{AB} \times \vec{AC}|}{2} = \frac{3 \cdot \sqrt{3}}{2}$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ 3 & 0 & 3 \end{vmatrix} = \begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix}$$

$|\vec{AB} \times \vec{AC}| = \sqrt{3 \cdot 9^2} = 3\sqrt{3}$

3.feladat

$$\underline{\underline{A}} \cdot \underline{v} = \underline{b} + 2\underline{v}$$

$$\left(\underline{\underline{A}} - 2\underline{\underline{E}}\right)\underline{v} = \underline{b}$$

$$\underline{v} = \left(\underline{\underline{A}} - 2\underline{\underline{E}}\right)^{-1} \cdot \underline{b}$$

$$\left(\underline{\underline{A}} - 2\underline{\underline{E}}\right) = \begin{pmatrix} 1 & -2 & 0 \\ 3 & -4 & 1 \\ 2 & 0 & 1 \end{pmatrix} \xrightarrow{\det} 1\cdot(-4) + 2\cdot(3-2) = -2$$

invertálható

$$\left(\underline{\underline{A}} - 2\underline{\underline{E}}\right)^{-1} = \frac{1}{-2}\begin{pmatrix} -4 & -1 & 8 \\ 2 & 1 & -4 \\ -2 & -1 & 2 \end{pmatrix}^T = \begin{pmatrix} 2 & -1 & 1 \\ 1/2 & -1/2 & 1/2 \\ -4 & 2 & -1 \end{pmatrix}$$

$$\underline{v} = \begin{pmatrix} 2 & -1 & 1 \\ 1/2 & -1/2 & 1/2 \\ -4 & 2 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1/2 \\ -6 \end{pmatrix}$$

4.feladat

$$\left(\begin{array}{ccc|c} 2 & 3 & -1 & 0 \\ 4 & 2 & -a & 0 \\ -1 & -2 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & -2 & 2 & 0 \\ 2 & 3 & -1 & 0 \\ 4 & 2 & -a & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & -2 & 2 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & -6 & 8-a & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & -2 & 2 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & -10-a & 0 \end{array}\right)$$

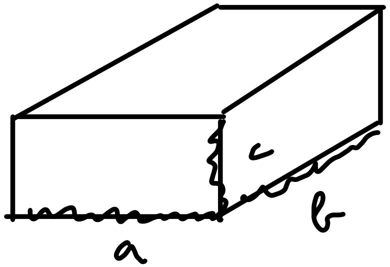
Ha $-10-a \neq 0$ azaz $a \neq -10 \Rightarrow$ 1 db mo $x = y = z = 0$

Ha $a = -10$ $\quad -y + 3z = 0 \quad\quad -x - 2y + 2z = 0$

∞ sok mo. $\quad y = 3z \longrightarrow -x = 2y - 2z = 6z - 2z = 4z$

$$z \in \mathbb{R};\ y = 3z;\ x = -4z$$

5.feladat



$a, b, c > 0$

$a + b + c = 30$

$V = a \cdot b \cdot c = ab(30 - a - b) = 30ab - a^2b - ab^2$

$V'_a = 30b - 2ab - b^2 = 0 \longrightarrow b(30 - 2a - b) = 0$

$b = 0$ nem jó; $\quad b = 30 - 2a$

$V'_b = 30a - 2ab - a^2 = 0$

Ha $b = 30 - 2a$

$30a - 2a(30 - 2a) - a^2 = 0$

$30a - 60a + 4a^2 - a^2 = 0$

$3a^2 - 30a = 0$

$3a(a - 10) = 0$

$a = 0$ nem lehet; $\quad a = 10 \rightarrow b = 10$

Stac. pont: $P(10, 10)$

$V''_{aa} = -2b \big|_{(10,10)} = -20$

$\det(\text{Hesse}(10,10)) = \begin{vmatrix} -20 & -10 \\ -10 & -20 \end{vmatrix} = 400 - 100 = 300 > 0$

van lok. sz. é.

$V''_{ab} = 30 - 2a - 2b \big|_P = -10$

$V''_{bb} = -2a \big|_P = -20$

$V''_{aa}(P) < 0 \Rightarrow a = b = 10 \; (= c)$

lokális maximumhelye V-nek.

6.feladat

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+n}}$$

Leibniz

$\Rightarrow$ konvergens

- alternáló $(-1)^n$ és $\sqrt{n^2+n} > 0$ miatt
- $\lim\limits_{n\to\infty} (-1)^n \cdot \dfrac{1}{\underbrace{\sqrt{n^2+n}}_{\to\infty}} = 0$
- $|a_n| = \dfrac{1}{\sqrt{n^2+n}}$

$$|a_{n+1}| \lesseqgtr |a_n|$$

$$\frac{1}{\sqrt{(n+1)^2+n+1}} \lesseqgtr \frac{1}{\sqrt{n^2+n}}$$

$$n^2+n \gtreqless n^2+3n+2$$

$$1 < 2n+1$$

monoton csökken $\Leftarrow$ $|a_{n+1}| < |a_n|$

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}} \qquad \frac{1}{\sqrt{n^2+n}} > \frac{1}{\sqrt{n^2+n^2}} = \frac{1}{\sqrt{2}\cdot n}$$

$\sum\limits_{n=1}^{\infty} \dfrac{1}{\sqrt{2}\cdot n}$ divergens minoráns sor (harmonikus sor)

A sor nem abszolút konvergens, de konvergens, ezért feltételesen konvergens.