1.feladat

$$z^4 - z^2 - 6z = 0$$

$$z(z^3 - z - 6) = 0 \Rightarrow z_1 = 0$$

osztói : $\pm 1; \pm 2; \pm 3; \pm 6$

$z_2 = 2 \quad 2^3 - 2 - 6 = 0$

$$z^3 - z - 6 : z - 2 = z^2 + 2z + 3$$

$$\begin{array}{l} \ominus\, z^3 - 2z^2 \\ \hline 2z^2 - z - 6 \\ \ominus\, 2z^2 - 4z \\ \hline 3z - 6 \\ \ominus\, 3z - 6 \\ \hline 0 \end{array}$$

$$z_{3,4} = \frac{-2 \pm \sqrt{4 - 4 \cdot 3}}{2} = -1 \pm \frac{\sqrt{-8}}{2} = -1 \pm \sqrt{2}\, i$$

$$z^4 - z^2 - 6z = z \cdot (z-2)(z + 1 + \sqrt{2}i)(z + 1 - \sqrt{2}i)$$

2.feladat

$$[\underline{a}\ \underline{b}\ \underline{c}] = \langle \underline{a} \times \underline{b} ; \underline{c} \rangle = \begin{vmatrix} 4 & 7 & -1 \\ 0 & 2 & 3 \\ 1 & -2 & -7 \end{vmatrix} = 4 \cdot \overset{-32}{(-14 + 6)} + 1 \cdot \overset{23}{(21 + 2)} = -9$$

$$V_{paral.} = \left| [\underline{a}\ \underline{b}\ \underline{c}] \right| = |-9| = 9$$

3.feladat

$$\underline{\underline{A}}\,\underline{x} = \underline{b}$$
$$\underline{x} = \underline{\underline{A}}^{-1}\cdot\underline{b}$$
$$\text{ha } \det \underline{\underline{A}} \neq 0$$

$$\det(\underline{\underline{A}}) = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -3 & 1 \\ 2 & 0 & -2 \end{vmatrix} = 2\cdot(-2-0)-2(-3+2) =$$
$$= -4+2 = -2$$

$$\underline{\underline{A}}^{-1} = \frac{1}{-2}\cdot\begin{pmatrix} 6 & -(-4) & 6 \\ -4 & (-2) & -4 \\ -2 & -1 & -1 \end{pmatrix}^{T} = -\frac{1}{2}\cdot\begin{pmatrix} 6 & -4 & -2 \\ 4 & -2 & -1 \\ 6 & -4 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 2 & 1 \\ -2 & 1 & 1/2 \\ -3 & 2 & 1/2 \end{pmatrix}$$

$$\underline{x} = \underline{\underline{A}}^{-1}\cdot\underline{b} = \begin{pmatrix} -3 & 2 & 1 \\ -2 & 1 & 1/2 \\ -3 & 2 & 1/2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -3+4-3 \\ -2+2-3/2 \\ -3+4-3/2 \end{pmatrix} = \begin{pmatrix} -2 \\ -3/2 \\ -1/2 \end{pmatrix}$$

4.feladat

$$\det(\underline{\underline{A}}-\lambda\underline{\underline{E}}) = \begin{vmatrix} 4-\lambda & 2 & 1 \\ 2 & 2-\lambda & -2 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 2\cdot(-4-(2-\lambda)) + (3-\lambda)((2-\lambda)(4-\lambda)-4) =$$
$$= -12+2\lambda+(3-\lambda)(\lambda^2-6\lambda+4) = -\cancel{12}+2\lambda+3\lambda^2-18\lambda+\cancel{12}-\lambda^3+6\lambda^2-4\lambda =$$
$$= -\lambda^3+9\lambda^2-20\lambda = \lambda(\lambda^2+9\lambda-20) = -\lambda(\lambda-4)(\lambda-5)$$
$$\lambda_1=0;\ \lambda_2=4;\ \lambda_3=5$$

$$\lambda_1 = 0: \begin{pmatrix} 4 & 2 & 1 \\ 2 & 2 & -2 \\ 2 & 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} 4 & 2 & 1 & 0 \\ 2 & 2 & -2 & 0 \\ 2 & 0 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 3 & 0 \\ 0 & 2 & -5 & 0 \\ 0 & 2 & -5 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 3 & 0 \\ 0 & 2 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \begin{array}{l} 2x + 3z = 0 \\ 2y - 5z = 0 \end{array} \quad \begin{array}{l} x = -\frac{3}{2} z \\ y = \frac{5}{2} z \end{array} \Rightarrow \underline{s} = \begin{pmatrix} -3z \\ 5z \\ 2z \end{pmatrix}; z \in \mathbb{R} \setminus \{0\}$$

$$\lambda_2 = 4: \begin{pmatrix} 0 & 2 & 1 \\ 2 & -2 & -2 \\ 2 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{l} 2y + z = 0 \rightarrow \\ x - y - z = 0 \\ 2x - z = 0 \rightarrow \end{array} \left\} \begin{array}{l} y = -z/2 \\ \frac{z}{2} - \left(-\frac{z}{2}\right) - z = 0 \checkmark \\ x = z/2 \end{array} \right.$$

$$\underline{s} = \begin{pmatrix} z \\ -z \\ 2z \end{pmatrix}; z \in \mathbb{R} \setminus \{0\}$$

$$\lambda_3 = 5: \begin{pmatrix} -1 & 2 & 1 \\ 2 & -3 & -2 \\ 2 & 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} -1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} -x + 2y + z = 0 \\ y = 0 \end{array} \right\} \begin{array}{l} -x + z = 0 \rightarrow x = z \\ y = 0 \end{array} \qquad \underline{s} = \begin{pmatrix} z \\ 0 \\ z \end{pmatrix}; z \in \mathbb{R} \setminus \{0\}$$

5.feladat

$$f(x,y) = \frac{xy}{x+y} \qquad f'_x(1,-2) = \frac{y(x+y)-xy}{(x+y)^2}\bigg|_{(1,-2)} = \frac{y^2}{(x+y)^2}\bigg|_{(1,-2)} = \frac{(-2)^2}{(-1)^2} = 4$$

$$\underline{\text{grad}}\, f(1,-2) = (4,1) \qquad f'_y(1,-2) = \frac{x(x+y)-xy}{(x+y)^2}\bigg|_{(1,-2)} = \frac{x^2}{(x+y)^2}\bigg|_{(1,-2)} = \frac{1^2}{(-1)^2} = 1$$

$$f'_\alpha(1,-2) = \left\langle (4,1), (\cos 135°, \sin 135°) \right\rangle = 4 \cdot \frac{-\sqrt{2}}{2} + 1 \cdot \frac{\sqrt{2}}{2} = -\frac{3}{2}\sqrt{2}$$

$$f(1,-2) = 1 \cdot (-2) \,/\, (-1) = 2$$

érintősík: $z = 4 \cdot (x-1) + 1 \cdot (y+2) + 2$

$$4x + y - z = 0$$

6.feladat

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$$

- alternáció $(-1)^n$ miatt $(n^2+1 > 0)$ ✓
- $\lim_{n\to\infty} \frac{(-1)^n}{n^2+1} = \frac{\pm 1}{\infty} = 0$ ✓
- $|a_{n+1}| \overset{?}{<} |a_n|$ mon. csökk.

$$\frac{1}{n^2+2n+2} < \frac{1}{n^2+1} \quad ✓$$

$$\cancel{n^2}+1 < \cancel{n^2}+2n+2$$

Leibniz-sor $\Rightarrow$ konvergens

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n^2+1} \right| = \sum_{n=1}^{\infty} \frac{1}{n^2+1} \qquad \frac{1}{n^2+1} < \frac{1}{n^2} \qquad \sum \frac{1}{n^2}$$

konvergens, majoráns
$p = 2 > 1$ hiperharmonikus

$\Rightarrow$ a sor abszolút konvergens