Matematika A2 Vizsga GTK NG&PSz MEGOLDÁSOK 2024.06.25.

1.feladat

$$\int_3^{\infty} \frac{1}{(2-x)^2}\,dx = \lim_{b\to\infty} \int_3^{b} \frac{1}{(2-x)^2}\,dx = \lim_{b\to\infty} \left[\frac{-1}{(2-x)} \cdot (-1) \right]_3^b =$$

$$= \lim_{b\to\infty} \frac{1}{\underbrace{(2-b)}_{\to -\infty}} - \frac{1}{(2-3)} = \frac{1}{-\infty} + 1 = 0 + 1 = 1$$

2.feladat

$$\alpha \cdot \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} + \gamma \cdot \begin{pmatrix} 1 \\ 3 \\ -6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{array}{ll} ① & 4\alpha + 2\beta + \gamma = 0 \\ ② & 2\beta + 3\gamma = 0 \\ ③ & 3\alpha - 2\beta - 6\gamma = 0 \end{array}$$

②-ből $\beta = -\frac{3}{2}\gamma$

③+② : $3\alpha - 3\gamma = 0$; $\alpha = \gamma$

① :

$$4\gamma + 2\left(-\frac{3}{2}\right)\gamma + \gamma = 0$$

$$5\gamma - 3\gamma = 2\gamma = 0$$

$$\Rightarrow \gamma = 0$$

Ha $\gamma = 0$, akkor $\alpha = \beta = 0$ $\Rightarrow$ csak az $\alpha = \beta = \gamma = 0$ komb. adja a nullvektort $\Rightarrow$ lineárisan függetlenek a vektorok

3.feladat

$$\left(\begin{array}{ccc|c} 3 & -1 & 3 & -1 \\ -2 & 1 & 2 & -1 \\ 4 & -1 & 8 & a \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 5 & -2 \\ -2 & 1 & 2 & -1 \\ 4 & -1 & 8 & a \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 5 & -2 \\ 0 & 1 & 12 & -5 \\ 0 & -1 & -12 & a+8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 5 & -2 \\ 0 & 1 & 12 & -5 \\ 0 & 0 & 0 & a+3 \end{array}\right)$$

Ha $a+3 \neq 0$ $(a \neq -3)$, akkor $\text{rang}(\underline{\underline{A}}) = 2 < \text{rang}(\underline{\underline{A}}|\underline{b}) = 3$

$\Rightarrow$ nincs megoldás

Ha $a+3=0$ $(a=-3)$, $\text{rang}(\underline{\underline{A}}) = \text{rang}(\underline{\underline{A}}|\underline{b}) = 2 < 3$ vált.sz.

$\Rightarrow$ ∞ sok megoldás, 1 szabad változó

$z \in \mathbb{R}$; $y + 12z = -5$; $x + 5z = -2$

$y = -12z - 5$ $\qquad x = -5z - 2$

4.feladat

$$\det\begin{pmatrix} 4-\lambda & -2 & 0 \\ 3 & -\lambda & -2 \\ 0 & 3 & -4-\lambda \end{pmatrix} = (4-\lambda)\Big(\underbrace{(-\lambda)(-4-\lambda)}_{4\lambda+\lambda^2} + 6\Big) - 3\Big(\underbrace{(-2)(-4-\lambda)}_{8+2\lambda}\Big) =$$

$$= 16\lambda + 4\lambda^2 + \cancel{24} - 4\lambda^2 - \lambda^3 - 6\lambda - \cancel{24} - 6\lambda = -\lambda^3 - 4\lambda =$$

$$= (-\lambda)\cdot(\lambda^2 - 4) = (-\lambda)(\lambda-2)(\lambda+2) \Rightarrow \lambda_1 = -2;\ \lambda_2 = 0;\ \lambda_3 = 2$$

$$\lambda_1 = -2 \qquad \begin{pmatrix} 6 & -2 & 0 \\ 3 & 2 & -2 \\ 0 & 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left. \begin{array}{l} 6x - 2y = 0 \rightarrow x = \frac{1}{3} y \\ 3x + 2y - 2z = 0 \\ 3y - 2z = 0 \rightarrow z = \frac{3}{2} y \end{array} \right\} \rightarrow$$

$$\rightarrow 3 \cdot \left(\frac{1}{3}\right) y + 2y - 2\left(\frac{3}{2}\right) y = y + 2y - 3y = 0 \checkmark \qquad \underline{s}_1 = \begin{pmatrix} 1/3\, y \\ y \\ 3/2\, y \end{pmatrix} = \begin{pmatrix} 2y \\ 6y \\ 9y \end{pmatrix} \quad y \in \mathbb{R} \setminus \{0\}$$

$$\lambda_2 = 0 \qquad \begin{pmatrix} 4 & -2 & 0 \\ 3 & 0 & -2 \\ 0 & 3 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{l} 4x - 2y = 0 \rightarrow y = 2x \\ 3x - 2z = 0 \rightarrow z = 3/2\, x \\ 3y - 4z = 0 \end{array}$$

$$\underbrace{y = 2x \quad z = 3/2\, x}_{3 \cdot 2x - 4 \cdot \frac{3}{2} x = 0 \checkmark}$$

$$\underline{s}_2 = \begin{pmatrix} x \\ 2x \\ 3/2\, x \end{pmatrix} = \begin{pmatrix} 2x \\ 4x \\ 3x \end{pmatrix} \quad x \in \mathbb{R} \setminus \{0\}$$

$$\lambda_3 = 2 \qquad \begin{pmatrix} 2 & -2 & 0 \\ 3 & -2 & -2 \\ 0 & 3 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left. \begin{array}{l} 2x - 2y = 0 \rightarrow x = y \\ 3x - 2y - 2z = 0 \\ 3y - 6z = 0 \rightarrow z = \frac{1}{2} y \end{array} \right\} \rightarrow$$

$$\rightarrow 3y - 2y - 2\left(\frac{1}{2} y\right) = 0 \checkmark \qquad \underline{s}_3 = \begin{pmatrix} y \\ y \\ \frac{1}{2} y \end{pmatrix} = \begin{pmatrix} 2y \\ 2y \\ y \end{pmatrix} \quad y \in \mathbb{R} \setminus \{0\}$$

5.feladat $f(x,y) = x^3 - 3xy + y^2 + y + 6$

$$\left.\begin{aligned} f'_x(x,y) &= 3x^2 - 3y \\ f'_y(x,y) &= -3x + 2y + 1 \end{aligned}\right\} \Rightarrow \begin{aligned} &3x^2 - 3y = 0 \rightarrow y = x^2 \\ &-3x + 2y + 1 = 0 \rightarrow -3x + 2x^2 + 1 \end{aligned}$$

Stac. pontok:

$$x_{1,2} = \frac{3 \pm \sqrt{9 - 4\cdot 2\cdot 1}}{4} = \frac{3 \pm 1}{4} \rightarrow \begin{matrix} 1/2 \\ 1 \end{matrix}$$

$P_1(1, 1)$

$P_2\left(\frac{1}{2}, \frac{1}{4}\right)$

$f''_{xx} = 6x \qquad f''_{xy} = -3 \qquad f''_{yy} = 2$

P_1: $\det(\text{Hesse}(1,1)) = \begin{vmatrix} 6 & 3 \\ -3 & 2 \end{vmatrix} = 12 - 9 > 0;\ f''_{xx} > 0 \Rightarrow$ lok. min.

$f(1,1) = 6$

P_2: $\det\left(\text{Hesse}\left(\frac{1}{2}, \frac{1}{4}\right)\right) = \begin{vmatrix} 3 & -3 \\ -3 & 2 \end{vmatrix} = 6 - 9 < 0$ nyeregpont

6.feladat

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} \cdot (x - (-3))^n \qquad \begin{aligned} a_n &= 1/[n\cdot(n+1)] \\ x_0 &= -3 \end{aligned}$$

$$\frac{1}{R} = \lim_{n\to\infty} \frac{1}{(n+1)(n+2)} \cdot \frac{n\cancel{(n+1)}}{1} = \lim_{n\to\infty} \frac{n}{n+2} = 1 \qquad R = 1$$

$\boxed{\text{Konv. tart.: } x \in [-4, -2]}$

ha $x_0 = -4$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} \cdot (-1)^n \rightarrow \text{Leibniz} \Rightarrow \text{konv.}$$

ha $x_0 = -2$

$$\sum_{n=1}^{\infty} \frac{1}{n\cdot(n+1)}$$

$\sum \frac{1}{n^2}$ konv. majoráns