VIZSGA 2023.06.07. MEGOLDÁSOK

1. feladat

$$z^4+3z^2-4=0$$

$$a=z^2$$

$$a^2+3a-4=(a+4)(a-1)=0$$ 2p

$a_1=-4$ $z_{1,2}^2=-4$ $a_2=1$ $z_{34}^2=1$

$z_{1,2}=\sqrt{-4}=\pm 2i$ 2p $z_{3,4}=\sqrt{1}=\pm 1$ 1p

Szorzatalak:

$$z^4+3z^2-4=(z-2i)(z+2i)(z-1)(z+1)$$ 1p

2. feladat

$$\underline{v} = (1, 2, 4)$$
$$\underline{a} = (3, 1, -1)$$

$$\underline{v}_{\parallel} = \frac{\langle \underline{v}, \underline{a} \rangle}{|\underline{a}|^2} \cdot \underline{a} = \frac{\langle (1,2,4), (3,1,-1) \rangle}{3^2 + 1^2 + (-1)^2} \cdot \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} =$$

1p

2p

$$= \frac{1 \cdot 3 + 2 \cdot 1 + 4 \cdot (-1)}{11} \cdot \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{11} \cdot \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3/11 \\ 1/11 \\ -1/11 \end{pmatrix}$$

$$\underline{v}_{\perp} = \underline{v} - \underline{v}_{\parallel} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 3/11 \\ 1/11 \\ -1/11 \end{pmatrix} = \begin{pmatrix} 8/11 \\ 21/11 \\ 45/11 \end{pmatrix}$$

2p

1p

3. feladat

$$\underline{\underline{A}} \cdot \underline{\underline{X}} = \underline{\underline{B}} \qquad \underline{\underline{A}} \in \mathbb{R}^{3\times 3} \quad \underline{\underline{B}} \in \mathbb{R}^{3\times 2} \Rightarrow \underline{\underline{X}} \in \mathbb{R}^{3\times 2}$$

$$\underline{\underline{X}} = \underline{\underline{A}}^{-1} \cdot \underline{\underline{B}} \quad \text{ha } \det(\underline{\underline{A}}) \neq 0$$ 1p

$$\det(\underline{\underline{A}}) = \begin{vmatrix} 4 & 0 & -1 \\ 0 & 2 & -1 \\ -4 & -1 & 2 \end{vmatrix} = 4 \cdot \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} - 1 \begin{vmatrix} 0 & 2 \\ -4 & -1 \end{vmatrix} = 4 \cdot 3 - 1 \cdot 8 = 4$$ 1p

$\underline{\underline{A}}$ invertálható

$$\underline{\underline{A}}^{-1} = \frac{1}{\det(\underline{\underline{A}})} \cdot \begin{pmatrix} \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} & -\begin{vmatrix} 0 & -1 \\ -4 & 2 \end{vmatrix} & \begin{vmatrix} 0 & 2 \\ -4 & -1 \end{vmatrix} \\ -\begin{vmatrix} 0 & -1 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} 4 & -1 \\ -4 & 2 \end{vmatrix} & -\begin{vmatrix} 4 & 0 \\ -4 & -1 \end{vmatrix} \\ \begin{vmatrix} 0 & -1 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 4 & -1 \\ 0 & -1 \end{vmatrix} & \begin{vmatrix} 4 & 0 \\ 0 & 2 \end{vmatrix} \end{pmatrix}^T = \frac{1}{4} \cdot \begin{pmatrix} 3 & 4 & 8 \\ 1 & 4 & 4 \\ 2 & 4 & 8 \end{pmatrix}^T = \begin{pmatrix} 3/4 & 1/4 & 1/2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

3p

$$\underline{\underline{X}} = \underline{\underline{A}}^{-1} \cdot \underline{\underline{B}} = \begin{pmatrix} 3/4 & 1/4 & 1/2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -3 & -2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ -1 & -1 \\ 1 & 0 \end{pmatrix}$$

1p 2p

4. feladat

$$f(x,y) = x^3 - x^2 - y^2 + y + 1$$

$$\left.\begin{aligned} f'_x(x,y) &= 3x^2 - 2x \\ f'_y(x,y) &= -2y+1 \end{aligned}\right\} \Rightarrow \begin{aligned} 3x^2 - 2x &= 0 \qquad x=0;\ x = 2/3 \\ -2y+1 &= 0 \Rightarrow y = 1/2 \end{aligned}$$

1p 2p

Stacionárius pontok: $P(0, 1/2)$ $Q(2/3, 1/2)$

Hesse-mátrix

$f''_{xx} = 6x-2$

$f''_{xy} = f''_{yx} = 0$

$f''_{yy} = -2$

1p

$$\det(\underline{\underline{H}}) = \begin{vmatrix} 6x-2 & 0 \\ 0 & -2 \end{vmatrix} = -12x+4$$

1p

$\det(\underline{\underline{H}})(P) = 4 > 0$ $f''_{xx}(P) = -2$

2p

lok. max. hely $f(P) = 5/4$

$\det(\underline{\underline{H}})(Q) = -4 < 0$

1p

nyeregpont, nincs sz.é.

5. feladat

a) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ pozitív tagú sor 1p

hányados kritérium

$\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = \lim_{n\to\infty} \frac{2}{n+1} = 0 < 1$ a sor konvergens (abszolút)

1p 1p 1p

b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ alternáló sor

abszolút sora $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ a majoráns 1p

kritérium alapján $\frac{1}{\sqrt{n+1}} > \frac{1}{n}$ ha $n > 1$ 1p

$\sum_{n=1}^{\infty} \frac{1}{n}$ divergens minoráns $\Rightarrow$ a sor <u>nem</u> abszolút konvergens 1p

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

- alternáló
- $\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = 0$
- $|a_n| = 1/\sqrt{n+1}$ monoton csökken

Leibniz-sor feltételesen konvergens 2p

6. feladat

$$f(x) = \frac{4+x}{3+x} = 1 + \frac{1}{3-(-x)} \quad \text{az} \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{ha } |x|<1$$

1p

alapján

$$1 + \frac{1}{3-(-x)} = 1 + \frac{1}{3} \cdot \frac{1}{1-\left(\frac{-x}{3}\right)} = 1 + \frac{1}{3} \cdot \sum_{n=0}^{\infty} \left(\frac{-x}{3}\right)^n =$$

2p

2p

$x_0 = 0$ körüli Taylor-sor

$$= 1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} \cdot x^n = \frac{4}{3} - \frac{1}{9}x + \frac{1}{27}x^2 - \frac{1}{81}x^3 + \dots$$

Konvergenciasugár $\left|\frac{-x}{3}\right| < 1$ $\quad |x| < 3$

1p

$R = 3$

2p