VIZSGA 2023.06.13. MEGOLDÁSOK

1. feladat (6p)

$$\int_0^3 \frac{2}{\sqrt{3-x}}\,dx = \lim_{b\to 3-} 2\cdot\int_0^b \frac{1}{\sqrt{3-x}}\,dx = \lim_{b\to 3-} 2\left[\frac{\sqrt{3-x}}{\frac{1}{2}}\cdot(-1)\right]_0^b =$$

(1p) (1p)

$$= \lim_{b\to 3-} -4\cdot\underbrace{\sqrt{3-b}}_{\substack{b\to 3-\\ \downarrow \\ 0+}} + \underbrace{4\sqrt{3-0}}_{4\cdot\sqrt{3}} = -4\cdot 0 + 4\sqrt{3} = \underline{\underline{4\sqrt{3}}}$$

(2p) (1p)

Az integrál konvergens. (1p)

2. feladat (8p)

$\underline{v}_1 = (2, -2, 0)$
$\underline{v}_2 = (1, 1, 3)$ } párhuzamos sík homálvektora

$$\underline{n} = \underline{v}_1 \times \underline{v}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -2 & 0 \\ 1 & 1 & 3 \end{vmatrix} = \begin{pmatrix} -6 \\ -6 \\ 4 \end{pmatrix}$$

ezzel párhuzamos $\underline{n}^* = (3, 3, -2)$ $|\underline{n}^*| = \sqrt{22}$ (2p)

$P(2, 0, -3)$ sík egyenlete:

$$S: 3(x-2) + 3y - 2(z+3) = 0$$

$$S: 3x + 3y - 2z = 12$$ (2p)

$Q(1, -1, 1)$ távolsága S-től

$$d(S, Q) = \left| \frac{3(1-2) + 3 \cdot (-1) - 2(1+3)}{\sqrt{22}} \right| = \left| \frac{-14}{\sqrt{22}} \right| = \frac{14}{\sqrt{22}}$$

(2p) (1p) (1p)

3. feladat (8p)

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix} \qquad \det(\underline{\underline{A}} - \lambda \underline{\underline{E}}_3) = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 4 & 2 & 1-\lambda \end{vmatrix} = (1-\lambda) \begin{vmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{vmatrix}$$

$$= (1-\lambda)\left((1-\lambda)^2 - 4\right) = (1-\lambda)(\lambda^2 - 2\lambda - 3) =$$

$$= (1-\lambda)(\lambda-3)(\lambda+1)$$

$\lambda_1 = 1 \quad \lambda_2 = 3 \quad \lambda_3 = -1$ (2p)

$\lambda_1 = 1$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} z = 0 \\ 4x + 2y = 0 \Rightarrow y = -2x \end{cases}$$

$$\underline{s} = \begin{pmatrix} x \\ -2x \\ 0 \end{pmatrix} \quad x \in \mathbb{R} \setminus \{0\}$$

(2p)

$\lambda_2 = 3$

$$\begin{pmatrix} -2 & 0 & 1 \\ 0 & -2 & 0 \\ 4 & 2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -2x + z = 0 \Rightarrow z = 2x \\ -2y = 0 \Rightarrow y = 0 \\ 4x + 2y - 2z = 0 \end{cases}$$

OK (2p)

$$\underline{s} = \begin{pmatrix} x \\ 0 \\ 2x \end{pmatrix} \quad x \in \mathbb{R} \setminus \{0\}$$

$\lambda_3 = -1$

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 4 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 2x + z = 0 \Rightarrow z = -2x \\ 2y = 0 \Rightarrow y = 0 \\ 4x + 2y + 2z = 0 \end{cases}$$

(2p)

$$\underline{s} = \begin{pmatrix} x \\ 0 \\ -2x \end{pmatrix} \quad x \in \mathbb{R} \setminus \{0\}$$

4. feladat (8p)

$V = 1\,m^3 = a \cdot b \cdot c \qquad a, b, c > 0 \qquad c = \frac{1}{ab}$ (1p)

$F(a,b,c) = 4 \cdot ab + 2bc + 2ac$ fedők kell

$f(a,b) = 4 \cdot ab + \frac{2}{a} + \frac{2}{b}$ (1p)

$f'_a = 4b - \frac{2}{a^2} \Rightarrow 4b - \frac{2}{a^2} = 0 \qquad \frac{1}{b} = 2a^2$ (2p)

$f'_b = 4a - \frac{2}{b^2}$ (1p) $\Rightarrow 4a - 2 \cdot (2a^2)^2 = 0 \qquad a - 2a^4 = 0$

$a = 0$, nem jó. $a > 0$

$a^3 = \frac{1}{2}$

$a = \frac{1}{\sqrt[3]{2}}$

Stac. pont $a = \frac{1}{\sqrt[3]{2}} \quad b = \frac{\sqrt[3]{4}}{2} = \frac{1}{\sqrt[3]{2}}$

$P(1/\sqrt[3]{2}, 1/\sqrt[3]{2})$

$f''_{aa} = \frac{4}{a^3} \qquad f''_{ab} = f''_{ba} = 4$ (1p)

$f''_{bb} = \frac{4}{b^3}$

$\det(\underline{\underline{H}}(P)) = \begin{vmatrix} 8 & 4 \\ 4 & 8 \end{vmatrix} = 64 - 16 > 0$ (1p) és

$f''_{aa} > 0 \Rightarrow$ lok. min. (1p)

$c = \sqrt[3]{4}$

5. feladat (7p)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

- alternáló
- $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(-1)^n}{\underbrace{(2n+1)!}_{\to\infty}} = 0$ (3p)
- $|a_{n+1}| < |a_n|$, mivel $\frac{1}{(2n+3)!} < \frac{1}{(2n+1)!}$

  $1 < (2n+2)(2n+3)$

A sor Leibniz - típusú így konvergens (1p)

Abszolút sora $\sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$ (1p) hányados-kritérium alapján

$$\lim_{n\to\infty} \frac{1}{(2n+3)!} \cdot \frac{(2n+1)!}{1} = \lim_{n\to\infty} \frac{1}{(2n+2)(2n+3)} = \frac{1}{\infty} = 0 < 1$$ (1p)

(1p)

tehát a sor abszolút konvergens is.

6. feladat (8p)

$$\sum_{n=1}^{\infty} \frac{x^n}{n^3 \cdot 4^n} \qquad x_0 = 0$$

$$a_n = \frac{1}{n^3 \cdot 4^n}$$ (1p)

konvergenciasugár:

$$\lim_{n\to\infty} \frac{1}{\sqrt[n]{n^3 \cdot 4^n}} \cdot |x| = \lim_{n\to\infty} \frac{1}{(\sqrt[n]{n})^3 \cdot 4} |x| = \frac{|x|}{4} < 1$$ (1p)

$|x| < 4 \quad R = 4$

$x \in (-4, 4)$-en biztosan konvergens (1p)

$$x = 4 \quad \sum_{n=1}^{\infty} \frac{4^n}{n^3 \cdot 4^n} = \sum_{n=1}^{\infty} \frac{1}{n^3}$$ (2p) konvergens, mert $\sum_{n=1}^{\infty} \frac{1}{n^p}$ konvergens, ha $p > 1$

$$x = -4 \quad \sum_{n=1}^{\infty} \frac{(-4)^n}{n^3 \cdot 4^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$$ (2p) abszolút sora konvergens, tehát a sor is.

A hatványsor konvergenciatartománya: $[-4, 4]$ (1p)