VIZSGA 2023.06.20. MEGOLDÁSOK

1. feladat (8p)

$z = 6i \qquad r = |z| = 6 \qquad \varphi = 90^\circ \qquad \sin\varphi = 1 \; ; \cos\varphi = 0$

1p 1p

$$6i = 6 \cdot \left(\cos(90^\circ) + i\sin(90^\circ)\right)$$

1p

$$\sqrt[3]{z} = \sqrt[3]{6\cdot\left(\cos(90^\circ) + i\sin(90^\circ)\right)} =$$

2p

$$= \sqrt[3]{6}\cdot\left(\cos\left(\frac{90^\circ + k\cdot 360^\circ}{3}\right) + i\sin\left(\frac{90^\circ + k\cdot 360^\circ}{3}\right)\right) \qquad k = 0,1,2$$

3p

$$z_1 = \sqrt[3]{6}\cdot\left(\cos(30^\circ) + i\cdot\sin(30^\circ)\right) = \sqrt[3]{6}\cdot\frac{\sqrt{3}}{2} + i\,\frac{\sqrt[3]{6}}{2}$$

$$z_2 = \sqrt[3]{6}\cdot\left(\cos(150^\circ) + i\sin(150^\circ)\right) = -\sqrt[3]{6}\,\frac{\sqrt{3}}{2} + i\,\frac{\sqrt[3]{6}}{2}$$

$$z_3 = \sqrt[3]{6}\cdot\left(\cos(270^\circ) + i\sin(270^\circ)\right) = -\sqrt[3]{6}\cdot i$$

2. feladat $(5p)$

$\underline{a} = (0, -1, 2)$

$\underline{b} = (2, 1, 3)$

$\underline{c} = (-2, 4, 1)$

$$[\underline{a}, \underline{b}, \underline{c}] = \begin{vmatrix} 0 & -1 & 2 \\ 2 & 1 & 3 \\ -2 & 4 & 1 \end{vmatrix} =$$

$(1p)$

$$= -(-1)\begin{vmatrix} 2 & 3 \\ -2 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ -2 & 4 \end{vmatrix} = 8 + 20 = 28$$

$(2p)$

$$V_{parall.pip.} = |28| = 28.$$

$(2p)$

3. feladat (8p)

$$\left.\begin{aligned} x + \quad\; z + 3u &= 5 \\ 2x + y \quad\; - u &= 10 \\ 3x + 2y + z + 2u &= 0 \end{aligned}\right\} \quad \left(\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 5 \\ 2 & 1 & 0 & -1 & 10 \\ 3 & 2 & 1 & 2 & 0 \end{array}\right) \begin{array}{c} \underline{s}_2 - 2\underline{s}_1 \\ \sim \\ \underline{s}_3 - 3\underline{s}_1 \end{array}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 5 \\ 0 & 1 & -2 & -7 & 0 \\ 0 & 2 & -2 & -7 & -15 \end{array}\right) \overset{\underline{s}_3 - 2\underline{s}_2}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 5 \\ 0 & 1 & -2 & -7 & 0 \\ 0 & 0 & 2 & 7 & -15 \end{array}\right)$$

3p

$\text{Rang}(\underline{A}|\underline{b}) < n = 4 \quad \infty$ sok mo. 1p

$$2z + 7u = -15 \implies z = \frac{-15}{2} - \frac{7}{2}u \qquad u \in \mathbb{R}$$

1p

$$y - 2z - 7u = 0 \implies y = 2z + 7u = -15 - 7u + 7u = -15$$

$$x + z + 3u = 5 \implies x = 5 - z - 3u = 5 + \frac{15}{2} + \frac{7}{2}u - 3u = \frac{25}{2} + \frac{1}{2}u$$

3p

4. feladat (7p)

$$f(x,y) = x\cdot e^y - \frac{5x^2}{y}$$

$$f'_x(x,y) = e^y - \frac{10x}{y}$$

$$f'_y(x,y) = x\cdot e^y + \frac{5x^2}{y^2}$$

(2p)

$P(1,1)$

$$\underline{\text{grad}}\, f(P) = \left(e^1 - \frac{10}{1} ;\ 1\cdot e^1 + \frac{5\cdot 1^2}{1^2}\right) = (e-10 ;\ e+5)$$

(2p)

Érintősík:

$$z = (e-10)\cdot(x-1) + (e-5)(y-1) + e-5$$

$$z = (e-10)x + (e-5)y - e + 10$$

(2p)

$$f(1,1) = 1\cdot e^1 - \frac{5\cdot 1^2}{1} = e-5$$

(1p)

5. feladat (4+5p)

a, $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$ pozitív tagú, gyökkritérium alapján (1p)

$$\lim_{n\to\infty} \sqrt[n]{\frac{2^n}{n^3}} = \lim_{n\to\infty} \frac{2}{\underbrace{(\sqrt[n]{n})^3}_{\to 1}} = 2 > 1$$ divergens. (2p) (1p)

b, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)}$ abszolút sora $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ (1p)

$\frac{1}{n^2+n} < \frac{1}{n^2}$ (1p) és $\sum_{n=1}^{\infty} \frac{1}{n^2}$ konvergens majoráns, (2p)

ezért az eredeti sor abszolút konvergens. (1p)

6. feladat (8p)

$f(x) = \frac{1}{2+x} = \frac{1}{2} \cdot \frac{1}{1-\left(-\frac{x}{2}\right)}$ függvény $x_0 = 0$ körüli Taylor-sora

1p

Mivel $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ $|x| < 1$-re ezért

2p

$$f(x) = \frac{1}{2} \cdot \frac{1}{1-\left(-\frac{x}{2}\right)} = \frac{1}{2} \sum_{n=0}^{\infty} \left(-\frac{x}{2}\right)^n = \frac{1}{2} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} \cdot x^n =$$

2p

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} \cdot x^n, \text{ ha } \left|\frac{x}{2}\right| < 1 \text{ azaz } |x| < 2$$

2p

a konvergenciasugár: 2.

1p