VIZSGA 2023.06.27. MEGOLDÁSOK

1. feladat (6p)

$z_1 = 3 - i$

$z_2 = 2 + 2i$

$$z_1 - 2 \cdot z_2 = 3 - i - 2(2+2i) = 3 - i - 4 - 4i =$$

$$= -1 - 5i \quad \text{2p}$$

$$\frac{z_1}{z_2} = \frac{3-i}{2+2i} = \frac{3-i}{2+2i} \cdot \frac{2-2i}{2-2i} = \quad \text{1p}$$

$$= \frac{(3-i)\cdot(2-2i)}{2^2 - 2^2 i^2} = \frac{(6 - 2i - 6i + 2i^2)}{8} = \frac{4-8i}{8} = \quad \text{1p}$$

$$= \frac{1}{2} - i \quad \text{2p}$$

2. feladat (6p)

$$\left.\begin{array}{l} A\,(2,2,0) \\ B\,(1,-1,1) \\ C\,(-3,0,4) \end{array}\right\} \quad \begin{array}{l} \vec{AB} = (-1,-3,1) \\ \vec{AC} = (-5,-2,4) \end{array}$$ (1p)

$$T_{ABC\Delta} = \frac{1}{2}\cdot\left|\vec{AB}\times\vec{AC}\right|$$ (1p)

$$\vec{AB}\times\vec{AC} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & -3 & 1 \\ -5 & -2 & 4 \end{vmatrix} = \begin{pmatrix} -12+2 \\ -(-4+5) \\ 2-15 \end{pmatrix} = \begin{pmatrix} -10 \\ -1 \\ -13 \end{pmatrix}$$ (2p)

$$\left|\vec{AB}\times\vec{AC}\right| = \sqrt{(-10)^2+(-1)^2+(-13)^2} = \sqrt{270} = 3\sqrt{30}$$ (1p)

$$T_{ABC\Delta} = \frac{3}{2}\cdot\sqrt{30}$$ (1p)

3. feladat (8p)

(2p)

$$\underline{\underline{A}} = \begin{pmatrix} 3 & 1 & 2 \\ -1 & p & -1 \\ 2 & 1 & 0 \end{pmatrix} \qquad \det(\underline{\underline{A}}) = 2 \cdot \begin{vmatrix} 1 & 2 \\ p & -1 \end{vmatrix} - 1 \begin{vmatrix} 3 & 2 \\ -1 & -1 \end{vmatrix} =$$

$$= 2(-1-2p) - (-3+2) = -1-4p$$

Ha $p = -\frac{1}{4} \Rightarrow \det(\underline{\underline{A}}) = 0$ a mátrix szinguláris (1p)

Ha $p = -1 \Rightarrow \det(\underline{\underline{A}}) = -1+4 = 3$ a mátrix invertálható (1p)

$$\underline{\underline{A}}^{-1} = \frac{1}{\det(\underline{\underline{A}})} \begin{pmatrix} \begin{vmatrix} -1 & -1 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} -1 & -1 \\ 2 & 0 \end{vmatrix} & \begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ -1 & -1 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ -1 & -1 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ -1 & -1 \end{vmatrix} \end{pmatrix}^T = \frac{1}{3} \cdot \begin{pmatrix} 1 & -2 & 1 \\ 2 & -4 & -1 \\ 1 & 1 & -2 \end{pmatrix}^T =$$

$$= \begin{pmatrix} 1/3 & 2/3 & 1/3 \\ -2/3 & -4/3 & 1/3 \\ 1/3 & -1/3 & -2/3 \end{pmatrix}$$

(4p)

4. feladat (8p)

$$f(x,y) = 3+x+y-3y^3-e^x$$

$$\left.\begin{aligned} f'_x &= 1-e^x \\ f'_y &= 1-9y^2 \end{aligned}\right\} \Rightarrow \begin{aligned} &1-e^x=0 \Rightarrow e^x=1 \Rightarrow x=0 \\ &1-9y^2=0 \Rightarrow y^2=\frac{1}{9} \Rightarrow y \; \pm\frac{1}{3} \end{aligned}$$

1p 2p

Stacionárius pontok $P(0,\frac{1}{3})$; $Q(0,-\frac{1}{3})$ 1p

$$f''_{xx} = -e^x$$

$$f''_{xy} = f''_{yx} = 0$$

$$f''_{yy} = -18y$$

1p

$$\det(\underline{\underline{Hesse}}(P)) = \begin{vmatrix} -1 & 0 \\ 0 & -\frac{18}{3} \end{vmatrix} = \frac{18}{3} > 0 \quad f''_{xx} < 0$$ 1p

P lok. max.

$$\det(\underline{\underline{Hesse}}(Q)) = \begin{vmatrix} -1 & 0 \\ 0 & \frac{18}{3} \end{vmatrix} = \frac{-18}{3} < 0$$ 1p

Q nyeregpont

$$f(0,\frac{1}{3}) = 3+0+\frac{1}{3}-3\cdot\frac{1}{3^3}-e^0 = 2\frac{2}{9}$$ 1p

5. feladat

a) (5p) $\sum_{n=1}^{\infty} \frac{n-2}{n^2(n+1)}$ ha $n>1$ pozitív tagú sor (1p)

$$\frac{n-2}{n^2(n+1)} < \frac{n}{n^3} = \frac{1}{n^2} \qquad \sum_{n=1}^{\infty} \frac{1}{n^2} \text{ konv. majoráns}$$

(1p) (2p)

$\Rightarrow$ eredeti sor is konvergens. (1p)

b) (4p) $\sum_{n=1}^{\infty} \frac{2^{n-1}+1}{4^n} = \sum_{n=1}^{\infty} \frac{1}{2}\cdot\left(\frac{2}{4}\right)^n + \left(\frac{1}{4}\right)^n$ (1p)

$\frac{1}{2}\cdot\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$ konv. geom. sor

$\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n$ konv. geom. sor (2p)

az eredeti sor is konvergens. (1p)

6. feladat (8p)

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{3\cdot n^2} \qquad x_0 = 2$$

$$a_n = \frac{1}{3n^2} \quad \text{1p}$$

Konvergenciasugár:

$$\lim_{n\to\infty} \left|\frac{(x-2)^{n+1}}{3\cdot(n+1)^2}\right| \cdot \left|\frac{3n^2}{(x-2)^n}\right| = |x-2| \lim_{n\to\infty} \frac{n^2}{(n+1)^2} = |x-2|\cdot 1 \;\; \text{2p} \; < 1$$

$|x-2| < 1 \Rightarrow R = 1$ a konv. sugár 1p

$-1 < x-2 < 1$

$1 < x < 3$ 1p

Ha $x = 1$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{3\cdot n^2} \Rightarrow$ absz. konv. 1p

$\Uparrow$

Ha $x = 3$ $\sum_{n=1}^{\infty} \frac{1}{3n^2} \Rightarrow \frac{1}{3}\cdot\sum_{n=1}^{\infty}\frac{1}{n^2}$ konv. 1p

Konvergenciaintervallum: $[1, 3]$ 1p